The Gilded Bubble Buffer*

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Abstract

We provide a microfounded framework for the welfare analysis of macroprudential policy by means of an overlapping generation model where productivity and credit supply are subject to random shocks in order to analyze rational bubbles that can be fueled by banking credit. We find that credit financed bubbles may be welfare improving because of their role as a buffer in channeling excessive credit supply and inefficient investment at the firms' level, but can cause systemic risk. Therefore macroprudential policy plays a key role in improving efficiency while preserving financial stability. Our approach allows us to compare the efficiency of alternative macroprudential policies. Contrarily to conventional wisdom, we show that macroprudential policy may be efficient even in the absence of systemic risk, that it has to be contingent on productivity shocks, to take into account real interest rates.

Keywords: Bank, bubble, macroprudential regulation

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"Beautiful credit! The foundation of modern society. Who shall say that this is not the golden age of mutual trust, of unlimited reliance upon human promises? "

–Mark Twain, The Gilded Age

1 Introduction

The recent empirical evidence has confirmed, once more, the existing connections between credit growth, leverage, bubbles and the likelihood of a banking crisis. Intuitively, it is clear that once a bubble starts, it explains both the credit growth and leverage because the bubble creates a demand for credit that is backed by the bubble itself as collateral. In this article we build a simple overlapping generation model in the tradition of Samuelson (1958), Diamond (1965) and Tirole (1985) that emphasizes the connections between the supply of credit, the equilibrium price of bubbles and systemic risk in the presence of banking. To the best of our knowledge, this is the first paper that models the link between rational bubbles, credit expansion and systemic risk has not been developed yet, in spite of the need to design a macroprudential policy that reduces the systemic risk inherent to credit expansion and the emergence of bubbles.

The explicit introduction of banks is of interest for three reasons: first, banks allow for levered bubbles; second, banks' solvency depends upon the behavior of the bubble, and third, a systemic crisis when bubbles are financed by banks may have much more devastating consequences than if bubbles are directly financed by investors, as the anecdotal evidence of the comparison between the dot-com bubble and the housing price bubble of 2007 shows, and the empirical analysis of Jordà et al. (2015) rigorously establishes.

There is now a large consensus on the impact of excessive credit growth on financial stability (Jordà et al., 2015; Laeven and Valencia, 2012; Schularick and Taylor, 2012) to the point that Schularick and Taylor (2012) conclude that "credit growth is a powerful predictor of financial crises". Still, even if the empirical literature points at excessive credit growth as one important correlate to financial crises, it is important to keep in mind that the amount of credit in an economy without financial innovations affecting the amount of loanable funds is endogenously determined, and so the question that arises is: how come there is both a sufficient amount of credit and a correlated amount of positive net present value projects? One answer is that deregulation and access to wholesale funds suddenly allowed to unleash credit and finance projects that were previously unable to obtain funding. The alternative is that the existence of credit may be a necessary condition for asset bubbles to emerge, so that, provided that the expected return of a bubble is sufficiently high, the demand for the bubble will create a demand for credit. This is an important point, as this implies that the supply of credit fuels bubble prices while a larger bubble fuels the demand for credit. So, in this market, the supply of credit creates its own demand.

From an empirical point of view, attributing the credit growth to the existence of bubbles is a complex exercise, as it requires defining a fundamental value and identifying the (positive) residual as the bubble. This will be difficult for real estate prices and may prove impossible for stock markets. It is no wonder that, prior to the 2008 financial crisis, the dominant view was that bubbles, if they ever exist could not be identified. Still, a number of contributions point out the importance of bubbles bursting as a cause of banking crises.[1] In particular, as Reinhart and Rogoff (2009) and Jordà et al. (2015) argue, credit-boom-fueled housing price spirals are particularly pernicious.

It is not surprising that the impact of bubble bursting on banks' solvency is of paramount importance when these bubbles have been financed by banks' credit. Japan and Scandinavian

[1] A first method to identify a bubble consists in detecting housing price deviations of real house prices above some specified threshold relative to trend (Borio and Lowe, 2002; Detken and Smets, 2004; Goodhart and Hofmann, 2008). A second approach focuses on the rate of growth of prices and diagnoses the existence of a bubble by the rate of growth being consistently above some threshold (Bordo and Jeanne, 2002). A third view identifies the bubble by the extent of the peak-trough (Helbling, 2005; Helbling and Terrones, 2003; Claessens et al., 2008). The different approaches can be combined to filter non-bubbles. This is why Jordà et al. (2015), while using the first approach of real increase relative to a trend, require also a subsequent price correction larger than 15%. A simpler, completely different strategy in identifying bubbles consists in analyzing the ratio of wealth to GDP, as the numerator, but not the denominator, might reflect the existence of a bubble. This is considered by Carvalho et al. (2012) who motivate their analysis by observing that the ratio of wealth to GDP grows before a crisis.

countries in the late 1980's are a clear example (Allen and Gale, 2000). Using a measure of bubbles that combines the trend of real estate prices with an ulterior price correction, Jordà et al. (2015) show that "when fueled by credit booms, asset price bubbles increase financial crisis risks". Similarly, Anundsen et al. (2014) recursively test whether credit and house prices are in a regime characterized by explosive behavior or not, and establish a positive and highly significant effect of exuberant behavior in the housing and credit market on the likelihood of a crisis.

From a theoretical perspective, the analysis of bubbles is particularly appealing because, first, it justifies simultaneously the bubble and the credit boom; second, it relates the bubble to the supply of credit, and, as a consequence, to saving gluts and capital mobility;[2] third, it makes systemic risk endogenous; and last, but not least, it frames the macroprudential policy in a set up where efficiency can be defined, so that the trade-off between economic growth and systemic risk is tractable.

Our research focuses on rational bubbles because this framework allows us to define the welfare properties of the equilibrium in a straightforward way.[3] In our model bubbles are assets that households purchase them in order to sell them one period later, but they yield no dividend nor are they used in production.[4] As in Martin and Ventura (2012), Caballero and Krishnamurthy (2006) and Doblas-Madrid (2012) bubbles are assets that are used as a savings vehicle. Contrarily to Farhi and Tirole (2012) and Martin and Ventura (2016), bubbles are not owned by entrepreneurs but by consumers, as bubbles constitute their best investment opportunity. Consequently, their positive role in resource allocation does not stem from their value in providing additional collateral to credit rationed firms but from preventing inefficient overinvestment and from providing a buffer as an automatic

[2] Justiniano et al. (2016) examine a number of possible explanations for the observed pattern of US housing prices and remark that the most current demand driven explanations are inconsistent with the decrease of the spread on mortgage rates, itself inconsistent with the increase (extensive margin) of riskier newcomers to the market; and that the critical turning point was the increase in private label securitization, quite in line with the results in Mian and Sufi (2009).

[3] See Brunnermeier and Oehmke (2013) for a complete perspective on bubble modeling in connection with systemic crises.

[4] Cryptocurrencies share similar characteristics.

mechanism to drain excessive liquidity or to supply an additional amount if required.

The analysis of bubbles requires to be specific about the alternative investment vehicles available to savers. We propose an overlapping generations (OLG) model where the transfer of goods from one generation to the next will be done through these vehicles. Our framework considers three different investment opportunities: acquiring the bubble, depositing in the bank and investing in a riskless asset. Because this asset is always available, in equilibrium, the real return on deposits and on bubble acquisition should be larger than the return on the riskless asset. Interestingly, in a fiat money economy, it seems natural to interpret the return on the riskless asset as the real return on holding cash, which constitutes a second bubble. Our results could then be viewed as the effect of the competition between two bubbles, with the possibility that one is relegated to its transactional role while the other provides a store of value role, as suggested in Tirole (1985).

Introducing banks that are confronted with stochastic shocks allows us not only to take into account the potential dynamic inefficiency of the economy, as in Diamond (1965) and Tirole (1985), but also to compute the macroprudential rules that maximize the expected aggregate domestic consumption. As we will see, some of the intuition for our results can be obtained from the allocation in a riskless steady state economy, but the analysis of bubble bursting and banks' bankruptcies requires the additional complexity of random shocks. We take these stochastic shocks to stem not only from productivity shocks but also from the liability side of banks, that affect their supply of credit. Focusing also on the liability side has the benefit of simplifying the analysis of the impact of capital regulation on the growth rate and riskiness of the resulting allocation. It allows also to consider the role of foreign capital inflows (or sudden stops), the creation of private money, as well as Central Banks liquidity injections or withdrawals.[5] Additionally, having a second source of shocks in the economy is consistent with financial shocks documented in Jermann and Quadrini (2012),

[5]Bernanke (2005) claims that the main reason for the current account deficits that US endured before the financial crisis was international capital flows. Our setup allows us to understand the effect of this type of flows.

among others. Still, obviously, our analysis holds in the absence of liquidity shocks with all the randomness driven by productivity shocks. We believe the introduction of liquidity shocks allows to obtain more interesting policy results in a model where the efficiency vs. systemic risk trade-off is tractable.

We obtain four regimes, depending on the relative scarcity of the credit supply, given the realized productivity shock: one where households deposit their savings at banks, a second one where households self-finance their purchases of the bubbles, a third one where households borrow from the bank to buy the bubble, and a fourth one where households save in the riskless asset.

We show, quite in line with Diamond (1965) and Tirole (1985), that, when the storage technology has a return lower than the exogenous growth of efficiency units, the efficient equilibrium will be the one characterized by leveraged bubbles. This is the case because, first, in the absence of a bubble, too low interest rates will lead to excessive capital accumulation causing dynamic inefficiency. Second, the equilibrium characterized by unlevered bubbles, where the return of the bubble is the same as the return on deposits, is also inefficient, because the spread between deposit and loan rates implies a high cost of capital for firms and a low level of investment, the opposite of what would happen in the levered bubbly equilibrium. Third, bubbles play a key role as a buffer by smoothing shocks in credit supply. Thus, quite in line with the empirical results (see for instance Laeven and Valencia (2012)), in our set up not all credit booms are inefficient.

Still, the existence of bubbles financed by bank loans leads to endogenous systemic risk. This is the case as a bubble bursting may lead to the bank bankruptcy with its subsequent cost for entrepreneurs to access funding. Consequently, our model allows us to examine the impact of macroprudential policies both on the efficiency of credit allocation and on the reduction of systemic risk.

Thus, our results are in line with what Allen and Gale (2000) document for Japan and

Scandinavia in the late 1980's and with the empirical finding in Jordà et al. (2015)[6] and with the conjectures put forward by Mishkin (2008), Mishkin (2009) and other policy- makers after the crisis: bubbles that threaten financial stability are those that are fueled by credit and leverage.

Our results coincide with Martin and Ventura (2015) in predicting that bubbles will emerge when interest rates are low and banking liquidity is high. In our framework this results in systemic risk, because the bubbles, even if held by household, will be financed by banks' credit. Macroprudential policy will then modify the equilibrium as it will change expectations on the future value of the bubble. If macroprudential policy is based on full information and is unconstrained, then it is expectationally robust as in their paper, in the sense that it isolates the economy from both liquidity and productivity shocks. Interestingly, macroprudential policy is useful even in the absence of systemic risk, since it helps bubbles to buffer shocks. Therefore the role of macroprudential policies might go beyond counter-acting externalities, complementing the externalities based justification of macroprudential regulation of De Nicolò et al. (2014) and Claessens (2014).[7]

Further, we find that it is welfare improving to set caps and floors on the total credit supply when households are borrowing to buy the bubble, as this reduces the riskiness of the economy, this improving allocation and reducing systemic risk. Still, when the only macroprudential policy is to impose caps on lending, this has the risk of lending to insufficient investment because of crowding out. Finally, we obtain that it is welfare improving to cap

[6]More precisely, the coefficient of the interaction between the credit variable and the bubble indicators is highly significant in determining the probability of a crisis, which fits our theoretical model.

[7]We focus on macroprudential policies, as opposed to, for instance, monetary policy. See Calza et al. (2013), Galí (2014) and Iacoviello (2005), among others, for an analysis of optimal monetary policy in an economy with bubbles. Nonetheless, our results speak to interactions between macroprudential policy and other economic policies: an expansive monetary policy that leads to more liquidity in the economy would have to be counteracted by a stricter macroprudential policy. Similarly, policies in a foreign country that affect international liquidity reaching the domestic economy also need to be counteracted by stricter macroprudential policies. Our approach differs from the monetarist classical view (Freeman and Haslag, 1996; Smith, 1991) in that their main objective is to identify the impact of interest payment on reserves on young and old generations, while in our approach it is the impact on firms' expected cost of capital that is crucial. In addition, our focus is on real interest rates that may be lower than the rate of growth, while in the classical view, the analysis focuses on the role of fiat money in an economy where a storage technology provides an exogenous rate of return.

total credit supply rather than capping targeted credit supply used to buy the bubble. While both regulatory mechanisms allow to cap the bubble and, therefore, systemic risk, capping targeted credit supply causes credit to flow towards firms, which causes marginal productivity of capital to reduce to inefficient levels. Consequently, our approach allows to highlight the benefit of countercyclical buffers rather than specific microprudential instruments, such as LTV or DTI aimed at curtailing the bubble.

This paper is not the first to address the issue of rational bubbles in their connection to systemic risk. Indeed, Aoki and Nikolov (2015) consider, as we do, bubbles in a banking economy and their role in generating banking crises. They show first, that when bubbles are bank-held, booms are amplified and crises are deeper. Second, they establish that when banks benefit from a generous deposit insurance scheme, are highly levered and long run interest rates are low (conditions that have prevailed before the 2008 crisis) they have higher incentives to hold bubbles. Still, our aim is broader as we explore the mechanism of bubble creation and bursting in connection with credit booms.

2 The Model

We will consider an overlapping generations economy with households, entrepreneurs and bankers that live for two periods. Entrepreneurs and households need financing and bankers have a role as providers of funds, because of their ability to screen, monitor and enforce contracts (as is Diamond, 1984). Absent banks, households may provide funding to firms but at a much higher cost that reduces the productivity of the economy. There is an exogenous cost of monitoring that implies that, in equilibrium, there is a spread between the bank offered deposit rate, r^d_{t+1} and its lending rate to firms r^f_{t+1} or households r^h_{t+1}.

2.1 Households

We assume that risk neutral households supply one unit of labor when young, and derive utility from their consumption only when old. We denote the consumption of generation i at time t by $c_{i,t}$. We make the usual assumption that households receive a labor endowment N_t when young, which allows them to obtain an equilibrium wage, and receive no endowment when old when they consume. For simplicity we take the labor supply to be inelastic and equal to $(1+n)^t$, where n denotes an exogenous rate of growth in efficiency units. Consequently, young households at time t save the equilibrium wage W_t in order to consume when old. They are able to do this either by depositing in the bank, by buying an asset in fixed supply with no return other than the price at which it will be sold in the future, which we refer to as the bubble, or by investing in a storage technology. We can interpret the bubble as a cryptocurrency with fixed supply: households will only buy it in order to sell it the following period. In contrast to Farhi and Tirole (2012) or Martin and Ventura (2016) it is households, not firms, that may invest in the asset that is susceptible of incorporating a bubble. Consequently, any effect that we may capture is completely unrelated to the role of collateral in providing better access to financial market.

The banking system allows households to either borrow an amount L_t at a rate r^h_{t+1}, to buy q_t of the bubble with price B_t or to deposit D_t at the bank with a return of r^d_{t+1}. We assume households cannot lend directly to firms, or, to be more precise, that they have a higher cost of screening and monitoring loans, in line with the idea that bankers emerge endogenously as the agents that are most efficient at granting loans. In addition, households can also save by investing O_t in a riskless asset, which offers an exogenous return that we denote by $\underline{r}$. It is important to notice that it is not possible for households to borrow at this rate.[8]

Consequently, households determine how to allocate their savings, between purchasing the bubble, depositing in the bank or investing in the riskless asset. Specifically, a household

[8] If the riskless asset is interpreted as a Treasury bill, only the Treasury can borrow at this rate.

born at t solves problem (1).

$$\max_{c_{t,t+1},q_t,D_t,L_t,O_t} \mathbb{E}_t c_{t,t+1} \tag{1}$$

$$\text{s. t. } q_t B_t + D_t + O_t \leq W_t + L_t$$

$$c_{t,t+1} \leq \max(0, q_t B_{t+1} - (1 + r^h_{t+1}) L_t + (1 + r^d_{t+1}) D_t + (1 + \underline{r}) O_t)$$

$$c_{t,t+1}, q_t, D_t, L_t, O_t \geq 0.$$

Notice that households are protected by limited liability that ensures $c_{t,t+1} \geq 0$.

2.2 Entrepreneurs

Each generation of entrepreneurs has a production technology and no other endowment. Similar to households, generation t of entrepreneurs lives at t and $t+1$ but consumes only at $t+1$.

We assume a representative firm. Its production process takes one period and, given a productivity level $\mathcal{A}_t$, allows to produce an output Y_{t+1} out of its inputs in labor and capital (N_{t+1}, K_t). K_t is borrowed at t, then labor is hired from generation $t+1$ and is paid out of the product Y_{t+1}, without requiring additional borrowing. The production process is simplified as we assume it derives from a riskless Cobb-Douglas production function. The output obtained is $Y_{t+1} = \mathcal{A}_t K_t^{\alpha} N_{t+1}^{1-\alpha}$, that , for the sake of simplification we take as riskless[9]The equilibrium in the labor market therefore determines the wage W_{t+1} at time $t+1$ as a function of the previous period capital K_t and the previous productivity shock $\mathcal{A}_t$. Because entrepreneurs live only two periods they fully consume their profits.

The problem entrepreneurs solve is the following:

[9] A random production would imply firms may default in their loans and systemic risk would be triggered by a combination of bubbles bursting and firms' bankruptcies without adding much to our result.

$$\max_{c^E_{t,t+1}, N_{t+1}, K_t} \mathbb{E}_t c^E_{t,t+1} \tag{2}$$

$$c^E_{t,t+1} \leq \mathcal{A}_t K_t^{\alpha} N_{t+1}^{1-\alpha} - W_{t+1} N_{t+1} - (1 + r^f_{t+1}) K_t$$

$$K_t \geq 0, N_t \geq 0, c^E_{t,t+1} \geq 0.$$

Because firms are price takers, W_{t+1}, is taken as given and it is not random as it depends upon the previous period $\mathcal{A}_t$ and K_t. We assume, for the sake of simplification, that capital fully depreciates in production. We can rewrite the aggregate profits of the firm as:[10]

$$\pi^F_{t+1} = (1+n)^{t+1} \left[\mathcal{A}_t \left(\frac{k_t}{1+n} \right)^{\alpha} - W_{t+1} - (1 + r^f_{t+1}) \frac{k_t}{1+n} \right],$$

where $k_t \equiv \frac{K_t}{N_t}$. This allows us to express interest rates relative to the exogenous growth n.

2.3 Bankers

Bankers are experts in screening, monitoring and enforcing payments on loans. This entails a payment to bankers to remunerate their task in monitoring and screening potential borrowers. Such cost implies a spread between deposit and loan rates.

Bankers live for two periods, are born with an endowment $\omega_t = \omega_0 (1+n)^t$, manage the bank when young and consume when old. They can use their endowment either to buy shares in an existing bank or to create a new bank. In either case, at the end of the period they consume the sum of the profits they obtain plus the value of the shares the next generation is buying, which is ω.

Banks' Funding

A bank obtains an exogenous amount of deposits S_t that could be interpreted as foreign

[10]In order to guarantee that entrepreneurs always have a positive consumption, we assume that the distribution of shocks in the economy has bounded support and that entrepreneurs have a constant endowment each period that is enough to guarantee that $c^E > 0$.

investment in the country, a central bank injection or a financial innovation that allows to obtain liquidity out of illiquid bank loans.[11] These funds depend on the overall financial structure of the economy, as they are related to the amount of private liquidity and to existing financial innovations and are remunerated at the same rate as the riskless asset, $\underline{r}$. This simplification means that the supply of funds to the economy is infinitely elastic up to the amount S_t. The generalization to a more realistic supply function would lead to some additional complexity, as S_t would be a function of the equilibrium interest rate r^f, without adding much to our results. Of course, if the supply function is deterministic, then the liquidity shocks disappear and the economy path is solely determined by productivity shocks.

The variable S_t plays a key role in the determination of the equilibrium as it is the basic determinant of the banks' supply of credit.[12] The introduction of a stock or bond market that finances a fraction of firms' investment does not affect the equilibrium, as it implies both a reduction of the demand for loans and the supply of savings.

We assume that, for regulatory reasons, banks cannot invest in bubbles, so that their investment choice is limited to lending to firms and to households. The balance sheet of the bank implies

$$K_t + L_t + O_t^B = E_t + S_t + D_t, \tag{3}$$

where O_t^B is the bank's investment in the riskless asset and E_t is the bank's equity before the t period profits Π_t are realized.

The bank's profits Π_t, that are generated by its operations at time t and accrue at the beginning of the next period, once B_{t+1} is known, are determined by:

[11] Although this assumption is restrictive, it is interesting to have some exogenous liquidity shock as a source of uncertainty. Adding some agents with different preferences and the ability to deposit in banks as a function of prevailing interest rates would add to realism but need not bring in additional insights.

[12] From that perspective, our reference to liquidity determined by exogenous foreign investment is a somewhat restrictive interpretation. Indeed, the amount of credit in the economy may also depend upon the type of collateral that banks accept in their credit operations. Buying stock on margin, as it was common before the 29 crisis, does increase banks' supply of credit. Credit default swaps that decrease the regulatory assessment of banks' credit risk and allow a higher banks' leverage, also increase the credit supply. Securitization also allows for an increase in credit supply.

$$\Pi_t = \min\left\{B_{t+1}, (1+r^h_{t+1})L_t\right\} - L_t + r^f_{t+1}K_t + \underline{r}(O^B_t - S_t) - r^d_{t+1}D_t$$
$$- \varphi(K_t + L_t), \tag{4}$$

where φ is the per dollar cost of financial intermediation, related to screening and monitoring loans, and the banks participation constraint is that the expected return on equity is at least equal to the deposit rate.

We assume that banks are price takers and, as a consequence, they offer any quantity of loans or deposits at the market rates. For the sake of simplicity we also assume depositors are insured at a zero insurance premium and that the random supply of liquidity S_t is inelastic.

The three interest rates the bank faces are related. First, the bank lends both to households and firms provided the expected return on the two types of loans the bank offers is the same. This implies that in any equilibrium where credit to households and firms is non zero, we have

$$\Pr\left(B_{t+1} \geq B^{SC}_{t+1}\right)(1+r^f_{t+1})L_t = \Pr\left(B_{t+1} \geq L_t(1+r^h_{t+1})\right)(1+r^h_{t+1})L_t + \tag{5}$$
$$+ \Pr\left(B^{SC}_{t+1} < B_{t+1} < L_t(1+r^h_{t+1})\right)\mathbb{E}(B_{t+1} \mid B^{SC}_{t+1} < B_{t+1} < L_t(1+r^h_{t+1})),$$

where B^{SC}_{t+1} is the level of the bubble for which a systemic crisis occurs. Equation (5) relates r^f_{t+1} to r^h_{t+1} and, as we will see, allows us to use only r^f_{t+1}. Notice that probabilities are conditional on $L_t > 0$, since otherwise r^h_{t+1} is not defined.

Second, under pure competition, the relationship between r^f_{t+1} and r^d_{t+1} is given by the cost of financial intermediation:

$$r^f_{t+1} = r^d_{t+1} + \varphi, \quad \varphi > 0. \tag{6}$$

This implies that $r^d_{t+1} < r^f_{t+1}$, so households will either borrow to buy the bubble or deposit, but will never do both at the same time.

The process for equity determines whether the bank is solvent or bankrupt. It depends upon the profits of the previous period, which are only realized once the price of the bubble B_t is obtained. Then,

$$E_{t+1} = \max\{0, E_t + \Pi_t - d_t\}, \tag{7}$$

where d_t is the dividend that is distributed. This means that the bank lends on the basis of the equity it has before profits, E_t and will only then incorporate its reserves $\Pi_t - d_t$ if accumulating reserves increases its consumption.[13]

The bank goes bankrupt at the end of time t whenever its losses are larger than its equity, $-\Pi_t > E_t$, in which case the dividend d_t will be restricted to be zero. The profit equation (4) shows that when banks' loans are repaid, profits are obviously positive, since $r^h_{t+1} > r^f_{t+1}$ $> r^d_{t+1}$ implies $\Pi_t > E_t r^d_{t+1}$. In our simplified context, where only the bursting of a bubble triggers a bank bankruptcy, a bank goes bankrupt whenever the bank's losses (from 4) are higher than its equity:

$$\Pi_t = B_{t+1} - L_t + r^f_{t+1}K_t + \underline{r}(O^B_t - S_t) - r^d_{t+1}D_t - \varphi(K_t + L_t) < -E_t,$$

so, a systemic risk occurs only if $L_t > 0$, and a necessary condition, as intuition suggests, is that the bank's losses on its bubble financing, $B_{t+1} - L_t$, are large enough; that is, if at period t the economy is in the $L-$regime, so that $O^B_t = D_t = 0$ and, in addition, B_{t+1} is small (i. e. the bubble has burst). Consequently, the probability of a systemic crisis is given

[13] Interestingly, if we consider the alternative formulation where banks lend before knowing their own capital and then find the required liquidity in the market, the supply of bank credit will determine the price of the bubble and make banks' profits indeterminate. This is in line with the joint determination of bank credit and bubbles' prices. Still, it means pushing the hypothesis of a representative bank too far, as the bank will then determine its own solvency.

by

$$\Pr(B_{t+1} < B^{SC}(E_t, S_t, \mathcal{A}_t)), \tag{8}$$

where $B^{SC}(E_t, S_t, \mathcal{A}_t) = L_t - r^f_{t+1}K_t + \underline{r}S_t + \varphi(K_t + L_t) - E_t,$.

The relationship between the level of the bubble, the credit supply and systemic risk is reminiscent of the results in Jordà et al. (2013), who examine the link between deregulation and credit growth and argue that a strong and sustained credit boom cannot be financed with local increase of deposits and wealth (especially if not driven by very strong fundamentals); foreign liquidity, or liquidity stemming from expansive monetary policy or financial innovation (e.g., securitization), needs to be present and to interact with the credit cycles. This is also confirmed by Dell'Ariccia et al. (2012) who show that, during a boom, imbalances build and the current account deteriorates.

Moral Hazard

To model the losses to society of a bank going bankrupt we assume that Banks' choice of their investment project in firms is subject to moral hazard, as in the classical Holmstrom and Tirole (1997) framework. They are able to implement a positive net present value project with constant returns to scale that returns y per dollar of investment and a negative net present value project that yields the same return but only succeeds with probability p_L ($p_L y < 1 < y$) and, in the adequate absence of adequate corporate governance, generates a private benefit U per dollar of investment. As a consequence, if R is the repayment to external liability holders per dollar of investment $\mathcal{L}$, the market will only finance banks if they have the incentives to implement the positive net present value project; that is, if $y - R \geq p_L(y - R) + U$, or

$$R \leq y - \frac{U}{\Delta p}, \tag{9}$$

where $\Delta p = 1 - p_L$ and $y - \frac{U}{\Delta p} < 1$, as otherwise the bank will be able to issue an infinite amount of external debt.

The maximum size of the project is $\mathcal{I}_\omega = \omega + S$, where S is the amount of external debt raised by the firm, so, if the repayment rate to the bank debt holders is $\underline{r}$, this implies $\mathcal{I}_\omega R = (1+\underline{r})(\mathcal{I}_\omega - \omega)$ and the incentive constraint (9) becomes:

$$\mathcal{I}_\omega \leq \alpha_L \omega,$$

where $\alpha_L = \frac{1}{1-\frac{y-\frac{U}{\Delta p}}{1+\underline{r}}}$, so that moral hazard determines a maximum size for the project, and

$$\mathcal{I}_\omega = \min(\alpha_L \omega, \omega + S).$$

Corporate Governance

We assume bank managers have the possibility to bring in external shareholders that improve corporate governance and reduce the extent of moral hazard by decreasing the value of private benefits from U to u. In order to do so, the bank has to issue external equity in some amount $\Gamma = E - w$, on which it pays a return $\underline{r} + \delta$, with $\delta > 0$. Because there is a per dollar non-pecuniary cost f of monitoring the bank, external shareholders monitor only if their per dollar return R_m is sufficiently high, so that $R_m \geq \frac{f}{\Delta p}$. Because equity is more expensive than external debt the above inequality will be binding and $R_m = \frac{f}{\Delta p}$.

Denoting by $\mathcal{I}_m$ the size of the project when the bank is monitored, the managers incentive constraint is given by

$$(1+\underline{r})(\mathcal{I}_m - E) + R_m \mathcal{I}_m \leq \left(y - \frac{u}{\Delta p}\right) \mathcal{I}_m,$$

implying $\mathcal{I}_m - E \leq \frac{1}{(1+\underline{r})}\left(y - \frac{u+f}{\Delta p}\right)\mathcal{L}$, so that for a given level of $E = \omega + \Gamma$,

$$\mathcal{I}_m \leq \alpha_H E$$

for $\alpha_H = \frac{1}{1-\frac{y-\frac{u+f}{\Delta p}}{1+\underline{r}}}$, which determines the maximum size of the project if the bank is not liquidity constrained.

Thus, if the bank submits to stricter corporate governance, its supply of credit is given by

$$\mathcal{I}_m = \min\left(\alpha_H E, S + E\right).$$

For $u + f << U$, $\alpha_H >> \alpha_L$ and the expected value of $\mathcal{I}_m$ will be much larger than the expected value of $\mathcal{I}_\omega$.

Banking Crises

When a bank goes bankrupt, a banking crisis occurs and the next generation of bankers creates a new bank. Still its commitment to a low level of private benefits is not credible because it has not yet external equity holders to impose sound corporate governance. This implies that the supply of credit shrinks from $\mathcal{I}_m$ to $\mathcal{I}_\omega$, which causes a loss to society.

Dividend Distribution and Capital Conservation

When the bank is solvent, only a fraction β belongs to the bankers' shareholders, with the rest belonging to the external shareholders that invested Γ. The dividend distribution policy is the one that maximizes the bankers' consumption, that will be the sum of the dividend received, βd_t, plus the share βP of the price P received from the sale of the bank to the next generation.

The next generation, that has an endowment ω, is willing to buy the bank provided this is not worse than the expected return from creating a new bank.

Consequently, the price of the bank will be such that bankers and new shareholders are willing to buy it: $\Gamma + \omega$ with the constraint $\beta(\Gamma + \omega) \geq V_\omega$, where V_ω is the expected value bankers obtain from creating a new bank. This constraint is clearly satisfied as it is the same that made it attractive for bankers to sell shares to external shareholders and benefit from a strict corporate governance.

As we know the value of the equity is given by $E_{t+1} = \max\{0, E_t + \Pi_t - d_t\}$, with $E_t = \Gamma + \omega$. Consequently, the bankers will set a dividend $\Pi_t = d_t$ if $\Pi_t \geq 0$ and sell the bank for the book value of its equity, $\Gamma + \omega$. Still, if the bank has suffered losses ($\Pi_t < 0$), for the new generation of bankers to accept buying the bank, the market price of the bank has to be reduced to $\Gamma + \omega + \Pi_t$, so that by investing $-\Pi_t$ the bank can continue operating with a sufficient recapitalization so as to keep capital equal to $\Gamma + \omega$. The only constraint is that the price is positive, $\Gamma + \omega \geq -\Pi_t$ which means the bank should not be bankrupt.

The implication is that, although the dividend flow is random, the value of equity at any point in time is

$$
\begin{aligned}
E_{t+1} &= \Gamma + \omega \text{ if } \Gamma + \omega \geq -\Pi_t \\
E_{t+1} &= \omega \text{ otherwise,}
\end{aligned}
$$

where the last equality is a result of the fact that a bankrupt bank can only use as equity the endowment of new bankers.

3 Quasi Stationary Equilibrium

Because the economy grows homothetically at the rate n, in the remaining of the paper we simplify the mathematical formulation and solve for $n = 0$, knowing that our results generalize. The equilibrium is quasi-stationary because for any given time-invariant distribution on $(S, \mathcal{A})$ it also depend upon the previous levels of equity and capital (E_{t-1}, K_{t-1}). It will be characterized by the expected future value of the bubble, $\mathbb{E}(B_{t+1})$ jointly with

$$\{r^f_{t+1}, r^h_{t+1}, r^d_{t+1}, q_t, N_t, W_t, K_t, L_t, D_t, B_t, E_t, q_t, c_{t-1,t}, c^E_{t-1,t}\}.$$

The stationarity of the distribution of shocks $(S, \mathcal{A})$, implies that the expected value of B_t depends only upon (E_{t-1}, K_{t-1}). As the labor supply is fixed, $N_t = 1$ and $q_t = 1$ and the variables E_t and W_t are determined by (E_{t-1}, K_{t-1}), we are left with six key variables,

$(r^f_{t+1}, K_t, L_t, D_t, B_t)$, that depend upon the realization of a state of nature $(S, \mathcal{A})$, while r^h_{t+1} and r^d_{t+1} derive from r^f_{t+1} through the cost of financial intermediation and credit risk (equations (5) and (6)).

In our framework, because $r^h > r^d$ and the fact that households cannot deposit at r^h or borrow at r^d, households cannot be both on the borrowing and on the deposit side. Therefore the expected return on the bubble may equal either the lending rate or the deposit rate. This already gives rise to two different regimes. In addition, as there is always the option of investing in the riskless asset, in some cases the deposit rate will equal the riskless rate. Finally, given that $\varphi > 0$, there might be states where households find r^f too high to borrow, but r^d too low to deposit.

As a consequence, four different regimes appear: an unlevered $U-$regime, where investors are indifferent between buying the bubble or depositing, a self financing regime (SF-regime) where household buy the bubble and neither borrow nor deposit, a levered $L-$regime, where households borrow to buy the bubble, and a $\underline{r}$ regime, where the interest rate on deposits is determined by the return on the riskless asset and depositors are indifferent between depositing in the bank or investing in the riskless asset.[14]

3.1 The Steady State Riskless Benchmark

The steady state certainty case provides an interesting benchmark to examine some of the characteristics of the equilibrium. These properties match the welfare properties of the equilibrium in the stochastic OLG equilibrium.

In a steady state the price of the bubble is constant. This implies that, in a bubbly regime, the return on holding the bubble is n. In an $L-$regime, the return of the bubble is the lending interest rate, so that $r^f = n$, and the equality between interest and growth rates that characterizes the Golden rule of the efficient allocations holds. On the other hand, in

[14]Notice that the balance sheet of the bank may shrink when we go from the $L-$regime to the $\underline{r}$ regime. This is in line with what happened in the US from 2001 to 2006 where banks' balance sheet expanded but shrank afterwards (Acharya and Naqvi, 2012).

the $U-$regime the return of the bubble equals the deposit interest rate, $r^d = n$, implying firms face a lending rate of $r^f > n$ that is higher than the growth rate. In the SF-regime, since the bubbles is self-financed, it must be the case that $r^f > n > r^d$. Finally in the $\underline{r}$ case, the bank has excess liquidity and it invests itself in the riskless asset and the return of the bubble equals $\underline{r}$.

Notice that while in Tirole (1985) there is an infinite number of bubbly equilibria, here we have only the one that is supported by the liquidity level S and productivity $\mathcal{A}$.[15] In contrast, in Tirole's classical framework the existence of a perfect market for funds implies there is always an endogenous level of liquidity that supports any bubble level.

3.2 Equilibrium Characterization

The Cobb-Douglas production function in problem (2) allows us to obtain explicit solutions for the stationary equilibrium for any given K_{t-1} and $E_{t-1}$and illustrate its main properties. For the sake of tractability, we now assume that $\alpha = \frac{1}{2}$. At time t, K_{t-1} is given and determines the demand for labor and the equilibrium wage W_t. S_t and $\mathcal{A}_t$ are the state variables, where S_t jointly with E_t determines $\mathcal{L}_t$.

We consider here the laws of motion of the different variables and abstract from the issue of banks' dividend distribution and capital accumulation that will be discussed later on. This allow us to consider the bank's supply of funds, which equals the sum of external funds deposited, S_t, and the bank accumulated equity, E_t, as the realization of a random variable whose distribution is known. We denote this liquidity by $\mathcal{L}_t = \min(\alpha_j E_t, S_t + E_t)$, with $j = L, H$, depending on the existence or not of strict corporate governance, so that the state of nature that defines the equilibrium is $(\mathcal{A}_t, \mathcal{L}_t)$.[16]

To begin with, in equilibrium, the demand for the bubble is derived from the first order condition of (1) with respect to q_t, which implies that the expected return from buying the bubble equals the interest rate (a condition sometimes referred to as "no arbitrage", a term

[15]To be strict, it is always possible to achieve an equilibrium with $B = 0$.

[16]Introducing an endowment shock as part of the state of nature is an obvious extension.

that does not apply *stricto sensu* in our framework).

Denoting the expected future price of the bubble by $\mathcal{E}_{t+1} = \mathbb{E}(B_{t+1})$ requires that $\frac{\mathcal{E}_{t+1}}{B_t} = 1+\underline{r}$ and $\frac{\mathcal{E}_{t+1}}{B_t} = 1+r^d_{t+1}$ in the $\underline{r}$ and U regimes, an equivalent expression, though complicated by the household limited liability,

$$\mathbb{E}\left(B_{t+1} \mid B_{t+1} \geq L_t(1+r^h_{t+1})\right) = (1+r^h_{t+1})B_t \tag{10}$$

for the $L-$regime and the following to hold for the SF-regime:[17]

$$1 + r^f_{t+1} > \frac{\mathcal{E}_{t+1}}{B_t} > 1 + r^d_{t+1}.$$

It is easy to show that, in equilibrium, equation (10) simplifies to $\mathcal{E}_{t+1} = (1+r^f_{t+1})B_t$, simply by multiplying by $\Pr(B_{t+1} \mid B_{t+1} \geq L_t(1+r^h_{t+1}))$ and using (5).

These conditions are quite similar to the "no-arbitrage" condition obtained in Farhi and Tirole (2012) as well as in Martin and Ventura (2016). Still, there is a critical difference, in that market segmentation, implying that households cannot borrow either at the deposit rate nor at the Treasury rate, makes this condition simply the result of households' maximization. It is therefore a marginal condition and in more general frameworks, e.g. with heterogeneous household, some of which have a utility for the bubble, the only requirement is that the first order condition holds for the **marginal investor.**

Existence of an equilibrium in this economy is related to the existence in Blanchard and Watson (1982) and Tirole (1985): the return on the bubble cannot be greater than n, since otherwise the bubble would outgrow the economy. Therefore, if $\underline{r} > n$ there is no bubbly equilibrium.

Notice that the equilibrium wage W_t is determined by the supply and demand equations

[17] An unexpected subsidy on interest rates resulting from a more favorable taxation or to cross-subsidization due to banking competition to attract clients will result in a higher price for the bubble. If the subsidy is permanent, it will then affect the expected value of the bubble as well as the current value.

$$\frac{\partial F(\mathcal{A}_{t-1}, N_t, K_{t-1})}{\partial N_t} = \frac{1}{2}\mathcal{A}_{t-1}K_{t-1}^{\frac{1}{2}}N_t^{-\frac{1}{2}} = W_t$$

$$N_t = 1,$$

so that it is determined from the conditions of the previous period. Additionally,

$$\frac{\partial F(\mathcal{A}_t, N_{t+1}, K_t)}{\partial K_t} = \frac{1}{2}\mathcal{A}_t K_t^{-\frac{1}{2}}N_{t+1}^{\frac{1}{2}} = 1 + r_{t+1}^f.$$

Consequently we can establish the characterization of the four different regimes, where the equilibrium conditions are a function of $\mathcal{A}, \mathcal{L}$ and $\mathcal{E} = E(B)$.[18] To define the equilibrium we first introduce the notation p_U, p_{SF}, p_L and $p_{\overline{r}}$ for the probabilities of the different regimes, depending on the distribution of $(\mathcal{A}, \mathcal{L})$. The expected value of the bubble is then:

$$\mathcal{E} = p_U \mathbb{E}(B^U) + p_{SF}\mathbb{E}(B^{SF}) + p_L \mathbb{E}(B^L) + p_{\underline{r}}\underline{B}, \tag{11}$$

where B^U, B^{SF}, B^L and $\underline{B}$ is the value of the bubble in the different regimes.

L-**regime**:

In the $L-$regime the laws of motion are defined by the following equations:

$$\begin{aligned} \mathcal{E} &= B(1+r^f) \\ K + L &= \mathcal{L} \\ B &= W + L \\ D &= O = 0, \end{aligned} \tag{12}$$

provided $L \geq 0$ and $r^f \geq \underline{r} + \varphi$. After replacing in $B = W + \mathcal{L} - K$ it is possible to solve

[18] To avoid cumbersome notation we drop the time subscript from the characterization. Still, it should be noticed that W is determined in the following period.

for the equilibrium values of $r^f(\mathcal{A},\mathcal{L},\mathcal{E})$, $B(\mathcal{A},\mathcal{L},\mathcal{E})$ and $K(\mathcal{A},\mathcal{L},\mathcal{E})$:

$$\begin{aligned}
r^f(\mathcal{A},\mathcal{L},\mathcal{E}) &= \frac{1}{2}\frac{1}{W+\mathcal{L}}\left[\mathcal{E}+\sqrt{\mathcal{E}^2+\mathcal{A}^2\left(W+\mathcal{L}\right)}\right]-1\\
K(\mathcal{A},\mathcal{L},\mathcal{E}) &= \frac{\mathcal{A}^2}{4(1+r^f(\mathcal{A},\mathcal{L},\mathcal{E}))^2}\\
B(\mathcal{A},\mathcal{L},\mathcal{E}) &= \frac{2}{\mathcal{A}^2}\mathcal{E}\left[\sqrt{\mathcal{E}^2+\mathcal{A}^2\left(W+\mathcal{L}\right)}-\mathcal{E}\right]\\
B(\mathcal{A},\mathcal{L},\mathcal{E}) &\geq W.
\end{aligned}$$

We can establish a boundary for $(\mathcal{A},\mathcal{L})$ so that the constraint $B(\mathcal{A},\mathcal{L},\mathcal{E})\geq W$ is satisfied. Using $W=\frac{2}{\mathcal{A}^2}\mathcal{E}\left[\sqrt{\mathcal{E}^2+\mathcal{A}^2\left(W+\mathcal{L}\right)}-\mathcal{E}\right]$, the constraint can be simplified to

$$\begin{aligned}
\mathcal{A} &\leq \mathcal{A}^L \equiv \mathcal{L}^{\frac{1}{2}}\frac{2\mathcal{E}}{W}\\
\mathcal{L} &\geq \mathcal{L}^L \equiv \left(\frac{\mathcal{A}W}{2\mathcal{E}}\right)^2.
\end{aligned}$$

To satisfy the $r^f(\mathcal{A},\mathcal{L},\mathcal{E})\geq \underline{r}+\varphi$ condition, the equivalent conditions are

$$\begin{aligned}
\mathcal{A} &\geq \mathcal{A}^{\underline{r}} \equiv 2\sqrt{\left(1+\underline{r}+\varphi\right)\left[\left(W+\mathcal{L}\right)\left(1+\underline{r}+\varphi\right)-\mathcal{E}\right]}\\
\mathcal{L} &\leq \mathcal{L}^{\underline{r}} \equiv \left(\frac{\mathcal{A}}{2(1+\underline{r}+\varphi)}\right)^2+\frac{\mathcal{E}}{1+\underline{r}+\varphi}-W.
\end{aligned}$$

SF**-regime**:

In the $SF-$regime the laws of motion are defined by the following equations:

$$
\begin{aligned}
1+r^f &> \frac{\mathcal{E}}{B} > 1+r^d \\
K &= \mathcal{L} \\
B &= W \\
D &= L = O = 0,
\end{aligned} \tag{13}
$$

which imply

$$
\begin{aligned}
K(\mathcal{A},\mathcal{L},\mathcal{E}) &= \mathcal{L} \\
r(\mathcal{A},\mathcal{L},\mathcal{E}) &= \frac{\mathcal{A}}{2\mathcal{L}^{\frac{1}{2}}} - 1 \\
B(\mathcal{A},\mathcal{L},\mathcal{E}) &= W.
\end{aligned}
$$

We can find the limits on $\mathcal{L}$ and $\mathcal{A}$ by using the first inequality that determines the laws of motion in this regime. Using the fact that $B = W$, $1 + r^f = \frac{\mathcal{E}}{W}$ results in $\mathcal{L}^L$ and $\mathcal{A}^L$, which are the boundaries that determine the L-regime. Therefore, the SF-regime occurs for lower values of $\mathcal{L}$ than the L-regime. By setting $1 + r^d = \frac{\mathcal{E}}{W}$ we get that the SF-regime occurs as long as

$$
\begin{aligned}
\mathcal{A} \leq \mathcal{A}^U &\equiv \mathcal{L}^{\frac{1}{2}} \frac{2\left(\mathcal{E}+\varphi W\right)}{W} \\
\mathcal{L} \geq \mathcal{L}^U &\equiv \left(\frac{\mathcal{A}W}{2\left(\mathcal{E}+\varphi W\right)}\right)^2.
\end{aligned}
$$

***U*-regime**:

In the $U-$regime the laws of motion are similar to the ones in the L-regime, but now the no arbitrage condition refers to the deposit rate and L becomes $-D$. The equilibrium is characterized by the following equations:

$$\begin{aligned} \mathcal{E} &= B(1+r^d) \\ K &= \mathcal{L} + D \\ B + D &= W \\ L &= O = 0, \end{aligned} \tag{14}$$

provided $D \geq 0$ and $r^d \geq \underline{r}$.[19]

Similarly to the L-regime, we obtain the $U-$regime solution as

$$\begin{aligned} r^f(\mathcal{A},\mathcal{L},\mathcal{E}) &= h_r^U(\mathcal{L},\mathcal{A},\mathcal{E}) \\ K(\mathcal{A},\mathcal{L},\mathcal{E}) &= \frac{\mathcal{A}^2}{4(1+r^f(\mathcal{A},\mathcal{L},\mathcal{E}))^2} \\ B(\mathcal{A},\mathcal{L},\mathcal{E}) &= h_B^U(\mathcal{L},\mathcal{A},\mathcal{E}) \\ B(\mathcal{A},\mathcal{L},\mathcal{E}) &\leq W, \end{aligned}$$

where h_r^U is strictly decreasing in $\mathcal{L}$ and strictly increasing in $\mathcal{A}$ and $\mathcal{E}$. h_B^U, on the other hand, is strictly increasing in $\mathcal{L}$ and $\mathcal{E}$ and strictly decreasing in $\mathcal{A}$. Similarly as before, we can establish a boundary for $(\mathcal{A},\mathcal{L})$ so that the constraint $B(\mathcal{A},\mathcal{L},\mathcal{E}) \leq W$ is satisfied.

$\underline{r}$-regime:

Finally, in the $\underline{r}-$regime the loan rate for the firm is equal to $\underline{r}+\varphi$ and the equilibrium is characterized by the following equations:

[19]In the case where the bubble is in real estate, a more cumbersome yet realistic model can be built, by assuming two classes of consumers, residential and financiers, with the former enjoying some utility from real estate while the latter not. This would imply that the price of real estate is larger than W even in the U-regime. Still, the no arbitrage condition holds only for the deposit rates.

$$
\begin{aligned}
\mathcal{E} &= B(1+\underline{r}+\varphi) \\
K+L+O^B &= \mathcal{L} \\
B &= W+L \\
D &= 0.
\end{aligned}
\tag{15}
$$

provided $L \geq 0$. From the first equation we can pin down B. From the household's budget constraint, we can pin down how much they borrow:

$$L = B - W.$$

Since we know r^f, we can determine how much the firms are borrowing, K. In this regime banks have extra liquidity that they are not able to lend. Therefore they invest in the riskless asset, O^B. From their balance sheet we can pin down this quantity:

$$O^B = \mathcal{L} - K - L.$$

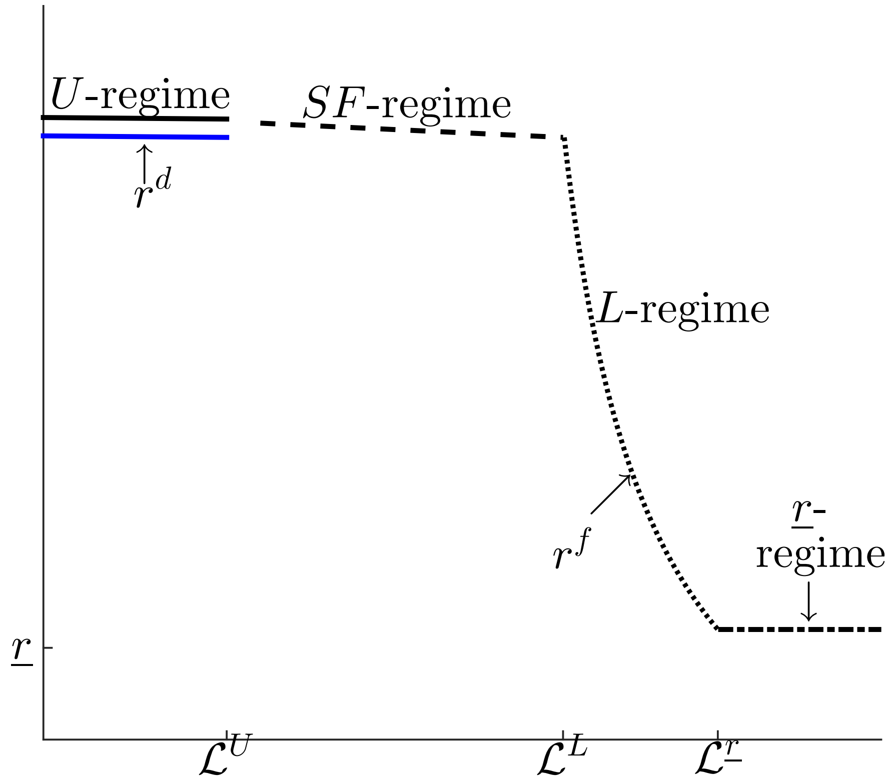


Figure 1: r^f and r^d

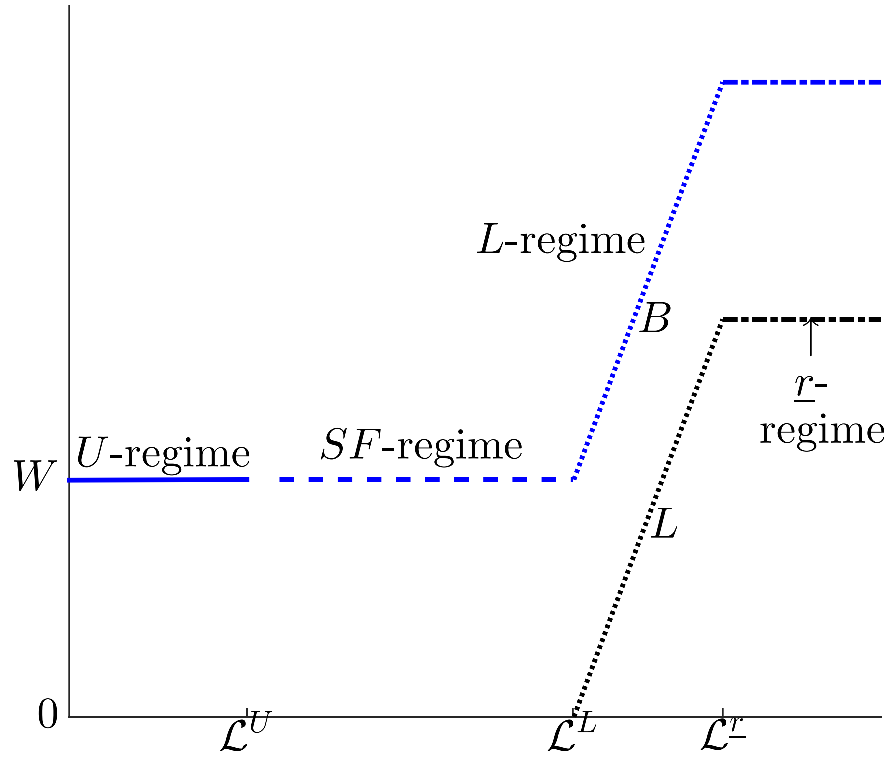
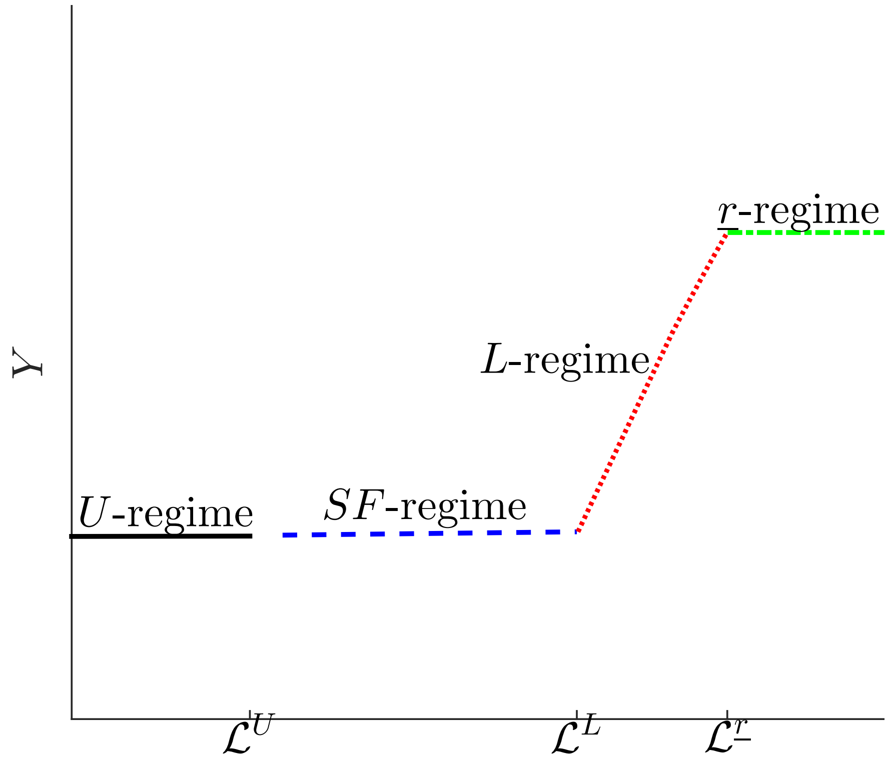


Figure 2: B and L



Y
$\underline{r}$-regime
L-regime
U-regime
SF-regime
$\mathcal{L}^U$
$\mathcal{L}^L$
$\mathcal{L}^{\underline{r}}$

Figure 3: Y

Figures 1 to 3 show different values across $\mathcal{L}$ assuming $F(0.5, N, K) = 0.5K^{\frac{1}{2}}N^{\frac{1}{2}}$ and assuming S follows a uniform distribution. Figure 1 illustrates the behavior of interest rates as a function of liquidity, with the line representing r^d being in blue and below the line showing r^f (in black). As intuition suggests, a liquidity shortage, will make interest rate soar and the U-regime prevails. This has an effect on the equilibrium price for the bubble as well as on the amount of credit that is channeled to the unproductive yet efficient investment, as shown in Figure 2. Figure 3 shows that production the following period is increasing in $\mathcal{L}$.

Notice that a bubbleless equilibrium exists with $B_t = 0$ and households investing in the riskless asset. Such an equilibrium implies $B_{t+1} = 0$, as otherwise there would be a demand for the bubble and a positive price.

3.3 The Equivalent Shocks Property

Our framework introduces two sources of uncertainty, liquidity and productivity. Still, it is easy to see that productivity shocks could play a symmetric role, as stated in the following proposition.

Proposition 1. *There exists intervals* $(\underline{\mathcal{A}}, \overline{\mathcal{A}})$ *and* $(\underline{\mathcal{L}}, \overline{\mathcal{L}})$ *such that for every positive productivity shock a negative liquidity shock exist with the same impact on the equilibrium interest rate and bubble price.*

Proof. See Appendix A. □

This property will be particularly helpful in providing additional guidance for the design of a macroprudential policy.

4 Welfare Properties

In defining welfare we have to take into account the welfare of the current as well as all future generations and, in particular, the risks they inherit of a bubble bursting. Because each generation consumes only when old, we cannot discount the different utilities, so we assume, as for instance Allen and Gale (1997) do, a planner that wants to maximize the long run average of the expected utilities of the different generations, given the starting capital K_0. This corresponds to the ex ante preferences of agents under the veil of ignorance whereby agents neither know ex ante whether they will be consumers, entrepreneurs or bankers and at what time they will be born. A utilitarian welfare function can then be defined as the expected per capita consumption, with expectations are taken given the probability distribution of $(\mathcal{A}_t, \mathcal{L}_t)$ conditional on all information relative to $t-1$:

$$\begin{aligned} V(K_0) &= \lim_{T\to\infty} \frac{1}{T} W_T(K_0), \text{ where} \\ W_T(K_0) &= \sum_{t=0}^{T-1} \frac{\left[\mathbb{E}_t c_{t,t+1} + \mathbb{E}_t c^E_{t,t+1} + \mathbb{E} c^B_{t,t+1} \mid K_0\right]}{N_t} \end{aligned}$$

Risk neutrality allows to simplify the problem, as it is possible to disregard the effect of redistribution between the three classes of agents and focus on the level of resources available for consumption in the economy.

At any time $t+1$, because repayment of loans and deposits are transfers between the bank and the agents, the aggregate goods available, $Y_{t+1} + S_{t+1} + (1+\underline{r})O^B_t$, are either consumed $(c^E_{t,t+1} + c_{t,t+1} + c^B_{t,t+1})$, invested in the production function as capital (K_{t+1}), or used to repay capital and interest to foreign investors, $(1+\underline{r})S_t$.[20] Consequently, we have $c_{t,t+1} + c^E_{t,t+1} + c^B_{t,t+1} = Y_{t+1} - K_{t+1} + \underline{r}O^B_t - \underline{r}S_t + (S_{t+1} - S_t) - (O^B_{t+1} - O^B_t) + \omega_t$.

As a result, the ex ante welfare associated to a random consumption $(c_{t,t+1}, c^E_{t,t+1}, c^B_{t,t+1})$ is equivalent, under our utilitarian assumptions, to the maximization of

$$\mathbb{E}\left[Y_{t+1} - K_{t+1} + \underline{r}O^B_t - \underline{r}S_t + (S_{t+1} - S_t) - (O^B_{t+1} - O^B_t)\right] + \omega_t,$$

[20] If instead of foreign investors it is the central bank that injects liquidity, then the central bank remuneration is redistributed and is part of the aggregate consumption.

with $Y_{t+1} = \mathcal{A}_t K_t^{\alpha} N_{t+1}^{1-\alpha}$ when banks are solvent and $Y_{t+1} = \mathcal{A}_t K_t^{\alpha} N_{t+1}^{1-\alpha} - \Delta Y$ in a banking crisis.

Denoting by $\widetilde{\Delta Y}$ the random variable that takes the value 0 in the absence of a crisis and ΔY when there is a crisis, then the equilibrium in each period is characterized by the realization of a random variable $\widetilde{S}_t$, which, in turn determines a realization for the equilibrium value of capital, $\widetilde{K}_t$. Specifically,

$$\mathbb{E}\left[\widetilde{\Delta Y_{t+1}}\right] = \Pr(B_{t+1} < B^{SC})\left[F(\mathcal{A}, N_{t+1}, K_t(\mathcal{L}_t)) - F(\mathcal{A}, N_{t+1}, K_t(\alpha_L \omega))\right].$$

Consequently,

$$W_T(K_0) = \sum_{t=0}^{T-1} \mathbb{E}\frac{1}{(1+n)^t}\left[F(\mathcal{A}_t, N_{t+1}, \widetilde{K}_t) - \widetilde{K}_{t+1} - \widetilde{\Delta Y}_t - (1+\underline{r})(\widetilde{S}_t - O_t^B) + (\widetilde{S}_{t+1} - O_{t+1}^B) + \omega_t\right]$$

and reordering the summations, we obtain

$$\begin{aligned} W_T(K_0) \quad = \quad & F(\mathcal{A}_0, N_1, K_0) - (1+\underline{r})(S_0 - O_0^B) \\ & + \sum_{t=1}^{T-1} \mathbb{E}\left[\frac{1}{(1+n)^t} F(\mathcal{A}_t, N_{t+1}, \widetilde{k}_t) - (1+n)\widetilde{k}_t - \frac{1}{(1+n)^{t+1}}\widetilde{\Delta Y}_{t+1} - \frac{1}{(1+n)^t}\underline{r}\widetilde{S}_t\right] \\ & + \mathbb{E}\frac{1}{(1+n)^{T-1}}\left[\widetilde{S}_{T+1} - O_{T+1}^B\right] + \omega_t, \end{aligned} \tag{16}$$

which reflects the trade-offs in costs and benefits of capital allocation. An increase in the marginal product of capital has an opportunity cost in terms of lost consumption and, also, in the systemic risk it might generates. This expression is interesting as it shows that the ex ante welfare depends exclusively on the allocation of capital; and that the efficient regime is characterized by Phelps' golden rule of equality between real interest rate and the rate of growth, as can easily be checked by taking the derivative with respect to k_t.

4.1 Inefficiency of the Bubbleless Equilibrium

With expression (16) it is then possible to assess the impact of the existence of bubbles on the equilibrium, which depends upon the regime.

Consider liquidity and productivity distributions that are ordered according to first order stochastic dominance, so that low levels and high levels of liquidity and productivity are well defined. Then the following proposition, whose proof is found in Appendix A, establishes the conditions under which the existence of a bubble improves efficiency.

Proposition 2. *For a given productivity shock $\mathcal{A}$, when liquidity is sufficiently high there exists a bubbly economy that is preferred to a bubbleless economy. Symmetrically, for a given liquidity level , when productivity is sufficiently low, there exists a bubbly economy that is preferred to a bubbleless economy.*

Remark 1. *The condition of sufficient liquidity, as it is intuitive, refers to the classical dynamic inefficiency condition, so that* $\mathbb{E}\left[\frac{\partial F(\mathcal{A}_t, N_{t+1}, \widetilde{\mathcal{L}_t + W_t})}{\partial k_t}\right] < 1+n$*. This condition is fulfilled either when the distribution of liquidity shocks is on average "sufficiently high" or when the distribution of productivity is on average "sufficiently low".*

Remark 2. *While the intuition is obvious and quite in line with the classical dynamic inefficiency results, the difficulty here is that the bubble is endogenous and if the amount of liquidity is only marginally high, the bubble may lead to insufficient capital accumulation.*

4.2 Inefficiency of the Bubbly Equilibrium

To assess the efficiency of the bubbly equilibrium, it suffices to check whether, on average, the golden rule is fulfilled.

The invisible hand argument does not work because the expected interest rate is larger than the rate of growth, so that the golden rule does not hold. To prove this point, notice, first that in each of the four regimes, it is the case that $\mathbb{E}(B) \geq \frac{\mathcal{E}}{1+r_t^f}$. Consequently, using (11), applying the law of iterated expectations and dividing by $\mathcal{E}$ leads to $\frac{1}{1+n} \geq \mathbb{E}\left[\frac{1}{1+r_t^f}\right]$, and because of the Jensen's inequality, $\mathbb{E}(1+r_f) > 1+n$. On average, the interest rate in the bubbly economy is excessively high. The result is not surprising and in line with the view that bubbles crowd out productive investment, which has a beneficial effect when liquidity is excessively high. It is only in the limiting case of a riskless economy that the equilibrium is efficient provided it is in the $L-$regime. This view can be expressed as stating that the riskiness of the economy has a cost, a point that is relevant in the analysis of the macroprudential policy.

5 Macroprudential Policy

Our stylized set up provides some insights for the time dimension in the analysis of macroprudential policy.[21] In our framework, two conditions are required for a banking crisis to occur. First, the economy has to be in the $L-$regime or in the $\underline{r}-$regime for the bubble to burst and cause a bank bankruptcy; second, the equity buffer of banks should be insufficient to cope with the losses the bubble bursting implies. Because, banks' bankruptcies are solely triggered by the bubble bursting, for a given level of banks' capital, the higher the bubble value, whether because of excessive credit or low productivity, the higher the systemic risk.

In order to model the macroprudential policy options, we denote by $m(\cdot)$ a function of the observable variables $\mathbf{X}(\mathcal{A},\mathcal{L})$ that affects the equilibrium outcome, so that the equilibrium depends now on both the random shocks and the macroprudential policy that is implemented[22] and may be partially contingent on the observable consequences of the shocks $(\mathcal{A},\mathcal{L})$. Computing its effects will, in general, be quite complex. Still, as mentioned, the utilitarian welfare function ignores redistribution among types of agents and through periods, so that the only relevant effect is through the level of capital, as can be seen in equation (16). In spite of this limitation, our specific assumptions allow us to derive some lessons

[21] If anything, the cross sectional dimension is reflected in the cost of a banking crisis as the banking industry characteristics, such as the level of interbank connections, complexity and the overall contagion affect this cost (Allen and Babus, 2009; Shin, 2010).

[22] The implementation of a macroprudential policy affects the whole equilibrium also through the change in the expected future value of the bubble.

regarding the efficient macroprudential policy, in particular that systemic risk is not just the result of excessive credit growth, but also of low productivity, an obvious but often neglected point, whether in measuring risk or implementing prudential policies.

Our analysis will proceed, first, by considering as a benchmark the first best where the macroprudential policy can be made contingent on the realized shock $(\mathcal{A}, \mathcal{L})$. Still, because the implementation of a contingent macroprudential policy may be informationally and institutionally excessively demanding, we extend our analysis to a second best framework where only the most classical macroprudential instruments will be considered.

5.1 Trade-offs in Macroprudential Policy

Current common wisdom implicitly posits a trade-off between the negative impact of macroprudential policy (in terms of capital allocation) on growth and the positive effect it has on systemic risk.

Contrarily to the well-received view that justifies the existence of a macroprudential policy solely on the cost of a systemic crisis, in our framework, macroprudential policy is justified even in the absence of systemic risk, as it may improve upon the unregulated equilibrium allocation. This is the case when there is capital overaccumulation, in which case no trade-off exists between efficiency and systemic risk mitigation. Indeed, a macro prudential regulation that reins in current growth by reducing investment might bring in a welfare increase in consumption. This is the case here because of the market incompleteness characteristic of rational bubbles. Thus, it would be incorrect to argue that macroprudential policy should be developed uniquely in order to correct negative externalities generated by systemic risk (Claessens, 2014; De Nicolò et al., 2012), except if we extend the concept of externalities so as to include the market incompleteness.

5.2 First Best Macroprudential Policies

As a starting point in the exploration of macroprudential policies, it is worth considering the case where regulatory authorities have perfect information and are unconstrained on the type of policy, so that both productivity and liquidity shocks are observable, and there is no constraint on the set of feasible macroprudential policies. These extreme assumptions allow for the fine tuning of the credit supply. As mentioned, because we focus on the longer term real effects of the policy, we simplify our analysis by considering monetary policy as determined by inflation while macroprudential policy allows to control the supply of credit. When a shock-contingent macroprudential policy can be defined, it is possible to decentralize the first best, provided the economy is in the $L-$regime, so that consumers that borrow in order to buy the bubble and firms face the same expected cost of borrowing, and at each point in time, t, the level of capital is such that the golden rule is fulfilled.

Proposition 3. ***(Golden Rule)*** *The full information unconstrained policy is characterized by a real interest rate in the $L-$regime equal to* 0 *(equal to n in an economy with resources growing at the rate n)*

The requirement to be in the L-regime is justified by the wedge between deposit and lending interest rates. It is only in the $L-$regime that households and firms are confronted with the same expected cost of capital.

As $\mathbb{E}(B_{t+1} \mid B_t) = \mathcal{E}_{t+1} = B_0(1+n)^t$, and $r^f = n$, in a quasi stationary economy it implies $\frac{\mathcal{E}_{t+1}}{N_{t+1}}$, $\frac{K_t}{N_t}$ and $\frac{B_t}{N_t}$ are constant. Notice that there is a multiplicity of efficient equilibria, as any B_0 sufficiently large so that the economy stays in the $L-$regime is compatible with the efficient allocation of capital. Because the macroprudential policy elminates any randomness on the bubble's price B_t, systemic risk is zero and, therefore, no solvency regulation on banks is required.

To implement such a policy, the regulatory authorities target an interest rate for firms that is equal to the growth in efficiency units and a constant value for $\frac{B_t}{N_t}$. The policy is better visualized in the case of no productivity shocks. The macroprudential policy implies that capital, the bubble and the supply of credit have to increase at the rate n. Consequently, for any exogenous liquidity supply $\mathcal{L}_t$, the macroprudential policy requires $m(\mathcal{L}_t) = \mathcal{L}_t^* - \mathcal{L}_t$, which is reminiscent of a sterilization policy, with a target $\mathcal{L}_t^* = (1+n)^t \mathcal{L}_0$, for $\mathcal{L}_0 = B_0 + K_0 - W_0$, and $\frac{\partial F(\mathcal{A},1,K_0)}{\partial k} = 1+n$.[23]

In the general case of productivity and liquidity shocks, productivity affects both K_t and W_t. Because in the first best $\frac{\partial F(\mathcal{A}_t,1,K_t)}{\partial k_t} = 1+n$, this allows to determine the optimal level of capital, $K_t^*(\mathcal{A}_t)$. Still, W_t is determined by the previous period productivity, $\mathcal{A}_{t-1}$, as it determines the marginal product of labor at time t, $\frac{\partial F(\mathcal{A}_{t-1},(1+n),K_{t-1})}{\partial N_t} = W_t(\mathcal{A}_{t-1})$.

From this expression and a bubble growing at a constant rate, it is possible to determine the optimal credit supply that is sufficient to finance both the optimal level of capital and the constant bubble:

$$\mathcal{L}_t^*(\mathcal{A}_{t-1}, \mathcal{A}_t) = K_t^*(\mathcal{A}_t) + B(1+n)^t - (1+n)W_t(\mathcal{A}_{t-1}, K_{t-1}),$$

so that the optimal macroprudential policy can be decomposed in two components, the liquidity adjustment, $m_1(\mathcal{L}_t) = \mathcal{L}_t^* - \mathcal{L}_t$ plus the productivity adjustment $m_2(\mathcal{A}_{t-1}, \mathcal{A}_t) = K_t^*(\mathcal{A}_t) + B(1+n)^t - (1+n)W_t(\mathcal{A}_{t-1}, K_{t-1})) - \mathcal{L}_t^*$.

The specification with Cobb-Douglas $\alpha = \frac{1}{2}$ and $n = 0$ allows to illustrate how the first best macroprudential policy operates. The optimal capital is $K_t^* = \frac{\mathcal{A}_t^2}{4}$. Consequently, from $\frac{1}{2}\mathcal{A}_{t-1} N_t^{-\frac{1}{2}} K_{t-1}^{\frac{1}{2}} = W_t$ we obtain $W_t = \frac{\mathcal{A}_{t-1}^2}{4}$. The optimal credit supply is $\mathcal{L}^*(\mathcal{A}_{t-1}, \mathcal{A}_t) = \frac{\mathcal{A}_t^2}{4} + B - \frac{\mathcal{A}_{t-1}^2}{4}$. The market credit supply depends on the liquidity shocks $\mathcal{L}_t$, so that the macroprudential adjustment of the credit supply $m(\mathcal{L}_t) = \frac{\mathcal{A}_t^2}{4} + B - \frac{\mathcal{A}_{t-1}^2}{4} - \mathcal{L}_t$ will take into account the liquidity shocks $B - \mathcal{L}_t$ and the productivity shocks $\frac{\mathcal{A}_t^2}{4} - \frac{\mathcal{A}_{t-1}^2}{4} - B$.

To sum up, two characteristics of the first best macroprudential policy should be highlighted. First, it is only possible to reach the efficient allocation in the $L-$regime; that is,

[23] Notice that such policy should take into account shadow banking insofar as it increases or destroys liquidity.

for a level of B larger than W that implies that households are levered. Second, the first best macroprudential policy is interventionist as it goes beyond the reduction of systemic risk and targets a constant interest rate.

5.3 Precommitted Macroprudential Policies

Because it is not always possible to attain a fully contingent first best allocation, we now turn to the comparison of the macroprudential policies that are set in advance and are unconditional on the effective shock realized. Notice that, form the present standpoint, the distinction between microprudential and macroprudential is irrelevant, as only the aggregate impact of these measures matters.

Our set up allows to consider the pros and cons of some macroprudential policies. Namely, we will focus on the effect of caps and floors on the aggregate amount of credit, and on the comparison between targeting the aggregate credit supply and targeting the fraction of credit that is used to buy the bubbly asset.

Namely, we will consider either limits on the total credit supply, $\mathcal{L}_t$, or on the loans to acquire the bubble, L_t. The first approach is related to Basel III countercyclical buffer, while the second is related to classical microprudential mechanisms, such as loan to value ratios, $L_t \leq \gamma_1 B_{t-1}$, or loan to income ratio $L_t \leq \gamma_2 W_t$. Because E_{t-1} is given at time t, it is equivalent to set these limits in terms of capital ratios $(E_{t-1}/\mathcal{L}_t)$. In general, both approaches pertain to policies aimed at counteracting prociclycality in an economy (Claessens et al., 2013; Claessens, 2014).

To analyze the impact of macroprudential policies it is helpful to start with a sufficient condition for a welfare increase.

Lemma 1. *A macroprudential policy that leads to a mean preserving contraction of K_t improves productive efficiency provided systemic risk is unchanged or decreased.*

Proof. As $F(\mathcal{A}, N_{t+1}, K) - K$ is concave in K, any mean preserving spread for $\widetilde{K}_t$ is welfare decreasing (Rothschild and Stiglitz, 1976). □

5.3.1 Caps and Floors On the Aggregate Credit Supply

The following Proposition shows that constraining the distribution of $\mathcal{L}$ might be welfare improving.

Proposition 4. *Imposing a cap $\overline{\mathcal{L}}$ and a floor $\underline{\mathcal{L}}$ on $\mathcal{L}$, with $\underline{\mathcal{L}}$ in the U or SF regime and $\underline{\mathcal{L}}$ in the L or $\underline{r}$ regime, such that $\mathbb{E}\left[\frac{\partial F(\mathcal{A}_t,N_{t+1},K(\mathcal{A},\underline{\mathcal{L}}))}{\partial k_t}\right] > 0$ and $\mathbb{E}\left[\frac{\partial F\left(\mathcal{A}_t,N_{t+1},K(\mathcal{A},\overline{\mathcal{L}})\right)}{\partial k_t}\right] < 0$ is welfare improving.*

Proof. See Appendix A. □

The existence of a cap clearly reduces systemic risk while the condition $\mathbb{E}\left[\frac{\partial F\left(\mathcal{A}_t,N_{t+1},K(\mathcal{A},\overline{\mathcal{L}})\right)}{\partial k_t}\right] < 0$ implies that, at the margin, productive investment has a negative net present value. Symmetrically, the existence of a lower bound allows to reduce the inefficiency of an excessively low volume of credit that would lead to insufficient productive investment. It is important to notice that, because caps and floors are defined in terms of the expected productivity shocks, the policy implies some ex post inefficiencies, as high productivity shocks may not be accommodated by an increase in liquidity once the cap is reached while low productivity shocks will not be dealt with through a decrease in the floor.

5.3.2 Cap On the Aggregate Credit Supply

When it is not feasible to impose a floor on the aggregate credit volume, the macroprudential policy will be based on setting only a cap on the credit supply. This correspond to the most frequent macroprudential policies, as solvency ratios, Basel III countercyclical buffers and limits to the credit to GDP ratio. The following proposition complements Proposition 4 and shows that such a macroprudential policy will be efficient as it limits the uncertainty associated with liquidity shocks.

Proposition 5. *Assume the distribution of $\mathcal{L}$ has its support in the $L-$regime. Then, for any given level of productivity, $\mathcal{A}$, imposing a cap $\overline{\mathcal{L}}(\mathcal{A})$ on $\mathcal{L}$ is Pareto efficient. A reduction of the cap $\overline{\mathcal{L}}(\mathcal{A})$ is Pareto efficient provided the economy stays in the $L-$regime.*

As in the general case of Proposition 4, the cost of imposing too strict a cap on the credit supply is that, with some probability, it leads to an inefficient capital allocation as it limits the ability for the economy to accommodate a productivity shock. Consequently, there is a trade-off between systemic risk and the risk of depriving highly productive firms from access to the credit market. Interestingly, macroprudential policy is useful even in the absence of systemic risk, since it helps bubbles to buffer shocks.

Proposition 5 has a limited scope, as its conditions may not be fulfilled if a cap leads to a lower expected value of the bubble and, therefore, to a higher probability of being in the SF or U regimes which implies a cost in terms of efficiency. Also, it considers a cap that is depending on $\mathcal{A}$ so that, once an unconditional cap is introduced it may lead to undershooting or to overshooting.

Note that in the L regime, our equilibrium is a mapping from $(\mathcal{A}, \mathcal{L})$ into (K, B, L, r_f), with its implications on wages W_t and banks' profits Π_t, so that imposing a cap $\overline{\mathcal{L}}(\mathcal{A})$, as stated in Proposition 5 changes the ex ante distribution of the states of nature, with $\mathcal{L}$ restricted to taking values lower or equal to the cap $\overline{\mathcal{L}}$. A simple interpretation would be to limit capital flows once the total credit volume reaches $\overline{\mathcal{L}}$. Notice that the equivalence between caps of volumes and tariffs leads to a reinterpretation in terms of taxing foreign capital.[24]

[24]Consider, a macroprudential policy that imposes a tax τ on foreign capital with its revenues redistributed back to banks as a lump sum. This would imply that the interest rate on firms' loans will satisfy: $r_f \geq \underline{r} + \tau + \varphi$. Now, for every cap $\overline{\mathcal{L}}$, when the economy is not constrained by banks' capital, there will be a tax on interest rates τ that will lead to the same equilibrium. Indeed, denote by $r_f(\overline{\mathcal{L}}, \mathcal{A})$ the level of interest rates that corresponds to $\overline{\mathcal{L}}$. In general, $r_f(\overline{\mathcal{L}}, \mathcal{A}) > \underline{r} + \varphi$, so that setting $\tau = r_f(\overline{\mathcal{L}}, \mathcal{A}) - \underline{r} + \varphi$, the equilibrium level (K, B, L, r_f) will be exactly the same.

Of course, the same policy absent the redistribution will lead to lower bank profits, possibly with an impact on the expected level of credit when the moral hazard constraint binds and, consequently on the expected level of the bubble, $\mathcal{E}$.

Notice that the above Propositions are related to capital requirements, but the countercyclical buffers emphasizes the role of credit to GDP, $\frac{\mathcal{L}_t}{F(\mathcal{A}_t, N_{t+1}, \widetilde{K}_t)}$. Thus, the ratio may reflect either a high productivity or an excessive capital investment in the last period and, in our set up, will lead to an inefficient amount of the credit supply at time t.

5.3.3 Comparing Caps on Credit Supply and Caps on Targeted Credit Supply

An alternative to the regulation of the credit supply is to regulate exclusively the segment of the credit supply that is used to buy the bubble. This would imply setting limits on the amount lent to households, so that $L \in (\underline{L}, \overline{L})$. For $\underline{L} > 0$, the equilibrium is always in the $L-$regime. As L is endogenously determined, a cap or a floor on this variable implies a segmentation in the credit market with different interest rates for consumer lending and for firms.

The following Proposition shows that it is better to cap total credit supply, which implies constraining loans to both firms and households, rather than capping targeted credit supply used to buy the bubble.

Proposition 6. *In the absence of systemic risk, for every cap on the targeted credit supply $\overline{L}$, there exists a cap on the total credit supply $\overline{\mathcal{L}}$ that is welfare increasing, provided $\overline{L}$ is such that the marginal product of capital is negative.*

Remark 3. *The proposition implies a trade-off between the efficiency in capital allocation and the reduction in systemic risk. Notice that this reduction is provided both by the cap on the amount lent and by the fact that banks are forced to provide a minimum of credit that supports the price of the bubble.*

To illustrate the results of this proposition, consider a limit on loan to value (LTV); that is, consider a limit $L \leq \lambda B$ for $\lambda \in [0, 1]$. Then there is a threshold on L, which we denote by L^λ, such that $L = \lambda B$ for $L \geq L^\lambda$. Since $L + W = B$, this implies that $L = \frac{\lambda}{1-\lambda} W$ and

$$\mathcal{L}^\lambda = \left(\frac{1}{1-\lambda} \frac{\mathcal{A}W}{2\mathcal{E}} \right)^2 + \frac{\lambda}{1-\lambda} W.$$

When the LTV constraint binds, $K = L - \frac{\lambda}{1-\lambda} W$. Therefore higher L translates to more capital and, therefore, a lower interest rate. Since $r^f \geq \underline{r} + \varphi$ there will also be a limit on $\mathcal{L}$ above which household are indifferent between borrowing to buy the bubble and buying the riskless asset. This limit is given by

$$\mathcal{L}^{\underline{r},\lambda} = \left(\frac{A}{2\left(1+\underline{r}+\varphi\right)}\right)^2 + \frac{\lambda}{1-\lambda}W.$$

We also consider a cap on $\mathcal{L}$ such that $\overline{\mathcal{L}} = \mathcal{L}^{\lambda}$. Under LTV, higher liquidity implies more resources flowing towards capital, since the limit on borrowing to buy the bubble binds. Figure 4 illustrated this. More capital implies higher production (Figure 5), beyond the point where marginal productivity is at the efficient level. As a result, even though production is increasing in $\mathcal{L}$, welfare becomes decreasing, as Figure 6 shows. On the other hand, a cap on $\mathcal{L}$ results in less resources going both to buy capital and to buy the bubble. As a result, systemic risk is still 0, but there is less capital misallocation.

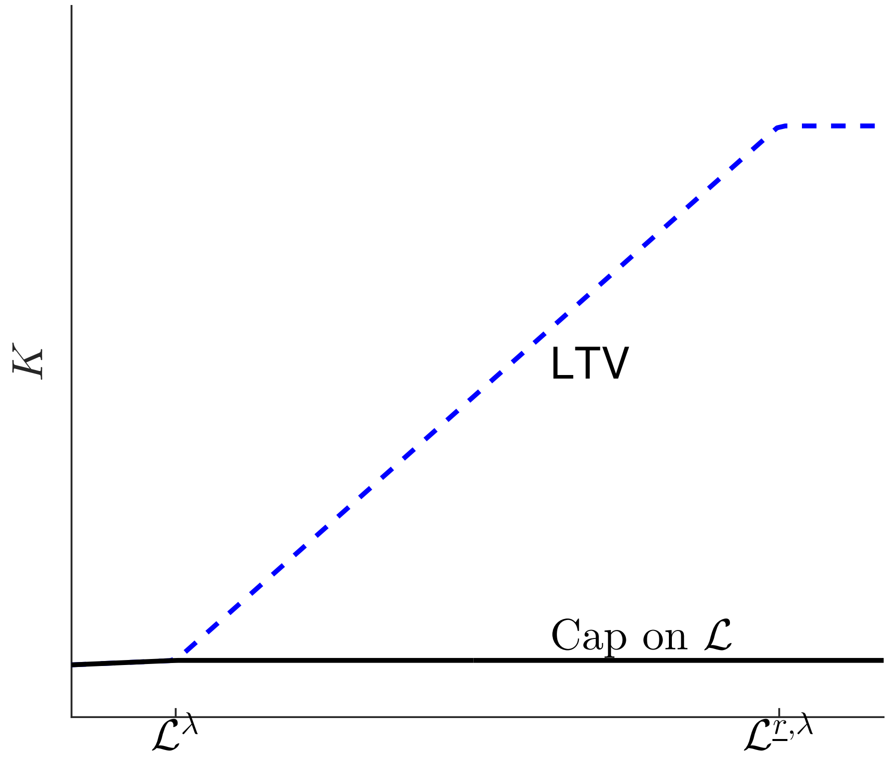


Figure 4: K for cap targeted credit supply and cap on total credit supply

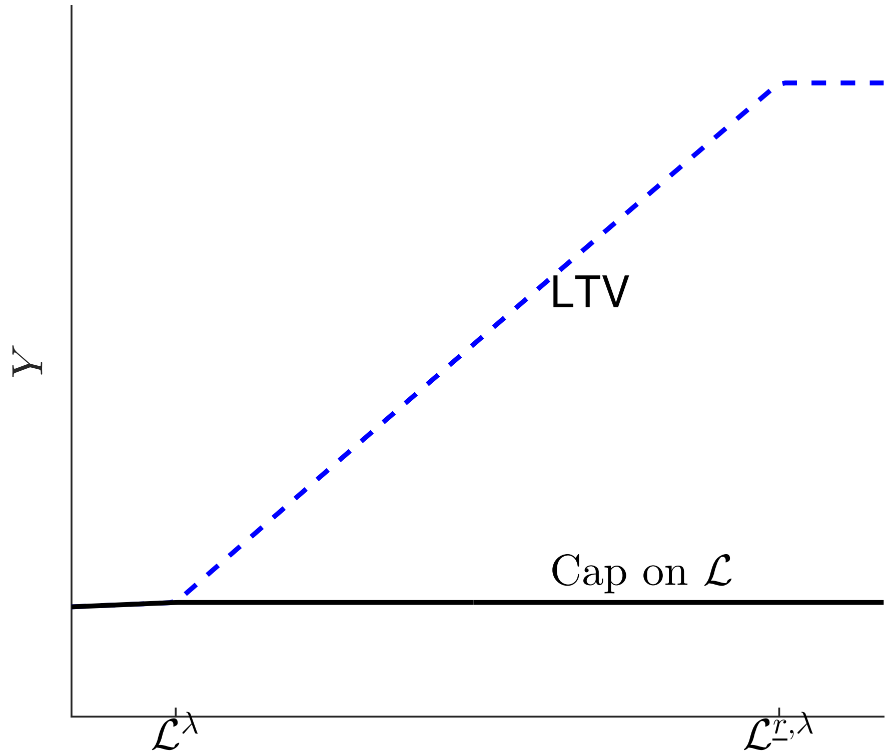


Figure 5: Y for cap targeted credit supply and cap on total credit supply

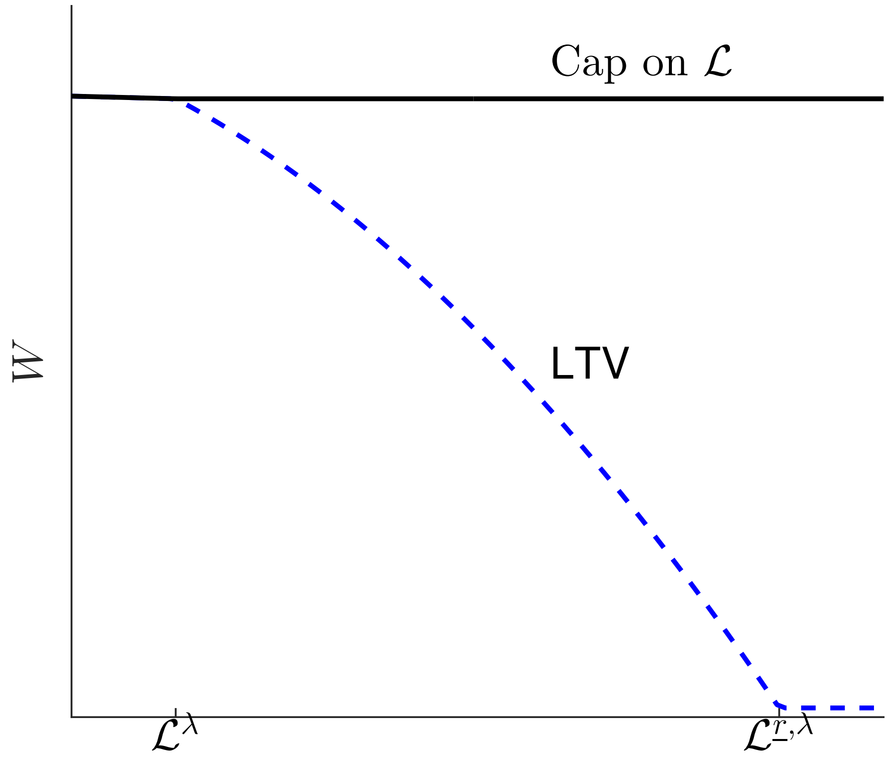


Figure 6: Welfare for cap targeted credit supply and cap on total credit supply

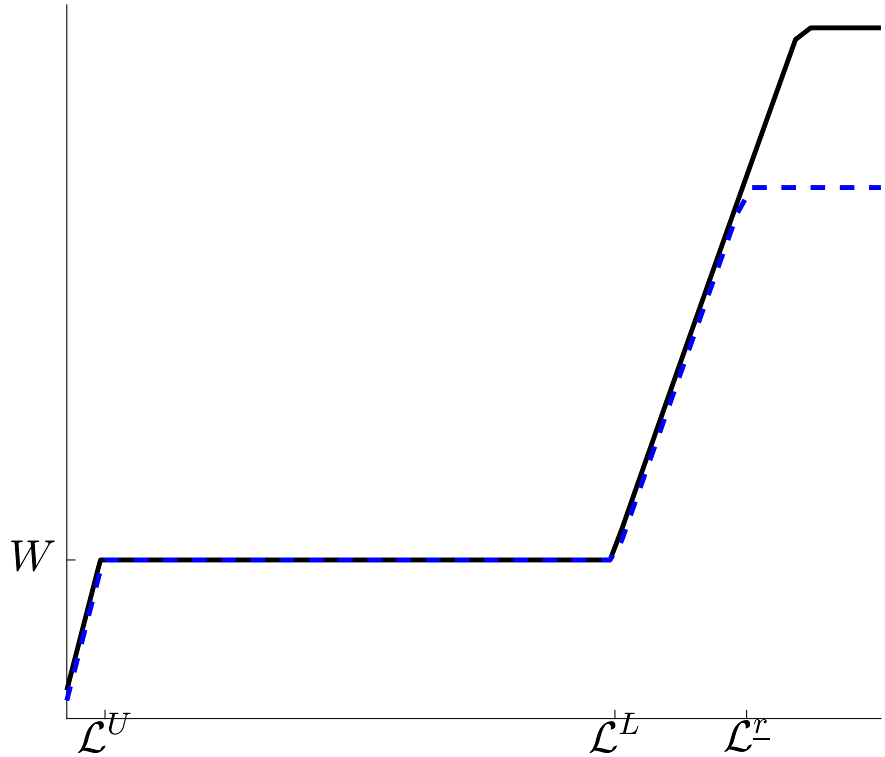


Figure 7: B when $\underline{r}$ changes

5.3.4 Increasing riskless rate

We also consider an alternative macroprudential policy: increasing $\underline{r}$. Such a policy directly decreases the threshold on $\mathcal{L}$ above which the $\underline{r}-$regime occurs and, since B is increasing in $\mathcal{L}$, also diminishes the value that the value attains in such regime. In equilibrium this policy will also have a negative an impact on the expected level of the bubble, $\mathcal{E}$.

In Figure 7 we show a numerical exercise comparing two equilibria. In this exercise the dotted blue line corresponds to an equilibrium with a higher value of $\underline{r}$. Even though increasing $\underline{r}$ augments the probability of occurrence of the $\underline{r}-$regime, the decrease in the value of the bubble is such regime compensate the increase in B risk and causes a lower $\mathcal{E}$ in equilibrium, which increases $\mathcal{L}^L$. As a result, a higher $\underline{r}$ decreases systemic risk.

6 Conclusion

Even if risky, rational bubbles constitute a way for households to transfer wealth from one period to another. This has a positive impact on the allocation of resources when, in the absence of bubbles, the financial markets do not allow to do so in an efficient way. As in the standard model, efficiency is reached when Credit, as "unlimited reliance upon human promises" has a role here because the efficient allocation of resources is reached for a unique interest rate, and this means households and firms face the same the return for their intertemporal transferring of resources. on the bubble should be the same as the return on productive investment. Still, in a banking economy, with different lending and a deposit rates,the efficient allocation is reached when bubbles are financed by credit. The other side of the coin is that if a bubble bursts it leads to bank losses that may wipe out the financial institution capital leading it systemic risk with its cost for society.

By modeling bubbles in an overlapping generation set up, it is possible to visualize the impact of credit, expectations formation and shocks on the resulting allocation. In equilibrium, the price of the bubble depends upon credit and upon the expected future price of the bubble. To some extent, the model shows that the supply of credit creates its own demand in so far as expectations react to the current prices.

Our analysis leads us to concentrate on two types of shocks, liquidity and productivity, that determine the equilibrium allocation. In this context, systemic risk is modeled as a bubble bursting when banks are insufficiently capitalized, an event that depend on these shocks as well as on the equilibrium price of bubbles.

Interestingly, this approach allows us to characterize macroprudential policy and, contrarily to conventional wisdom, it is possible to show that macroprudential policy should consider the type of shock that occurs: reacting to a liquidity shock or to a productivity shock implies different macroprudential policies. Also, systemic risk depends upon the productivity shocks, as we show that a lower productivity leads to a higher price for the bubbly asset and a higher systemic risk. Additionally, macroprudential policy is useful even in the absence of systemic risk, since it helps bubbles to buffer shocks. Finally, we find that it is welfare improving to impose a cap and a floor on total credit supply rather and than capping the total credit supply dominates capping the targeted credit supply used to buy the bubble.

Of course, for each stochastic process that the exogenous shock follows, (stationary, martingales or , mean reverting,...) there is a different view of bubbles. Nevertheless, this characteristic of the analysis should not be seen as a limitation, but rather as the necessity for policy makers to be well aware of the nature of these shocks.

Appendix A Mathematical Appendix

Proof of Proposition 1

We restrict the proof to the L-regime, since an equivalent result holds for the U-regime. To avoid cumbersome notation, we abstract from time subscripts and we refer to r^f as r.

It is sufficient to prove that in the resulting equilibrium r is monotonically increasing in the shock $\mathcal{A}$ and monotonically decreasing in the shock $\mathcal{L}$. Since F is strictly concave in K and strictly increasing in $\mathcal{A}$, then demand for credit, $K(r, \mathcal{A}) = \left(\frac{\partial F(\mathcal{A},1,K)}{\partial K}\right)^{-1}(r)$, is strictly decreasing in r and strictly increasing in $\mathcal{A}$.

In the L-regime we have

$$B^L = W + L = W + \mathcal{L} - K(r, \mathcal{A}) \geq \underline{B}.$$

Additionally, from the first equation in (12) we can derive

$$\mathcal{E} = (1 + r)(W + \mathcal{L} - K(r, \mathcal{A}))$$

which implicitly determines $r(\mathcal{L}, \mathcal{A})$ that satisfies

$$\frac{\partial r}{\partial \mathcal{L}} = \frac{1}{W + \mathcal{L} - K(r, \mathcal{A}) - (1 + r)\frac{\partial K}{\partial r}} > 0$$
$$\frac{\partial r}{\partial \mathcal{A}} = \frac{(1 + r)\frac{\partial K}{\partial \mathcal{A}}}{W + \mathcal{L} - K(r, \mathcal{A})} > 0.$$

Proof of Proposition 2

For a bubbly equilibrium to exist, we need $r_f \leq n$. For $\widetilde{\mathcal{L}_t}$ high enough the bubbly economy is in the L-regime. Consider a succession of economies characterized by a liquidity distribution $\widetilde{\mathcal{L}_t}$, such that, for some $\overline{\mathcal{L}_t}$, we have i) $\widetilde{\mathcal{L}_t} \leq \overline{\mathcal{L}_t}$; ii)$\lim_{t\to\infty} \Pr(\widetilde{\mathcal{L}_t} = \overline{\mathcal{L}_t}) = 1$. Then, $\overline{\mathcal{L}_t}$ will also be in the L-regime or in the $\overline{r}-$regime, so that firms interest rates equal the expected return on the bubble. The economies converge to the riskless bubble that is characterized by $r_f = n$, a bubble growing at the rate n and zero systemic risk. Consequently, for t sufficiently large, the bubbleless economy is characterized by dynamic inefficiency while the bubbly one is close to efficiency.

Proof of Proposition 4

In the absence of any cap and floor, the total average expected welfare is given by

$$W = \int_0^\infty \int_0^\infty (F(\mathcal{A}, N, K(\mathcal{A}, \mathcal{L})) - K(\mathcal{A}, \mathcal{L}))d\mathcal{H}(\mathcal{A} \mid \mathcal{L})d\mathcal{G}(\mathcal{L}) - \mathbb{E}_\mathcal{A}\Delta Y(\mathcal{A}, \mathcal{L})),$$

and, denoting by

$$\Psi(\mathcal{L}) = \mathbb{E}_\mathcal{A}\left[(F(\mathcal{A}, N, K(\mathcal{A}, \mathcal{L})) - K(\mathcal{A}, \mathcal{L})\right],$$

the first part, that corresponds to the allocation, can be written as $W^A = \int_0^\infty \Psi(\mathcal{L})d\mathcal{G}(\mathcal{L})$.

$\Psi(\mathcal{L})$ is concave and has a maximum at $\mathcal{L}^*$ that corresponds to the first order condition $\mathbb{E}\left(\frac{\partial F(\mathcal{A},N,K(\mathcal{A},\mathcal{L}^*))}{\partial k}\right) = 1+n$.

We will first consider the impact of the introduction of caps and floors on total lending in the absence of systemic risk ($\mathbb{E}_{\mathcal{A}}\Delta Y(\mathcal{A},\mathcal{L})) = 0$) and compute the difference $\Delta = W^A - W^A_{CF}$ in welfare that results from the imposition of the caps and floors, where W^A is welfare without constraints on $\mathcal{L}$ and W^A_{CF} is welfare with the cap and the floor on $\mathcal{L}$. Specifically,

$$W^A_{CF} = \mathcal{G}(\underline{\mathcal{L}})\Psi(\underline{\mathcal{L}}) + \int_{\underline{\mathcal{L}}}^{\overline{\mathcal{L}}} \Psi(\mathcal{L})d\mathcal{G}(\mathcal{L}) + (1-\mathcal{G}(\overline{\mathcal{L}}))\Psi(\overline{\mathcal{L}}).$$

Consequently, Δ can be written as

$$\Delta = \int_0^{\underline{\mathcal{L}}} (\Psi(\mathcal{L}) - \Psi(\underline{\mathcal{L}}))d\mathcal{G}(\mathcal{L}) + \\ + \int_0^{\underline{\mathcal{L}}} (\Psi(\mathcal{L}) - \Psi(\overline{\mathcal{L}}))d\mathcal{G}(\mathcal{L})$$

As $\mathcal{L}^*$ is the maximum and the function is concave, we have $\Psi(\mathcal{L}) - \Psi(\underline{\mathcal{L}}) < 0$ for any $\mathcal{L} < \mathcal{L}^*$. Similarly $\Psi(\mathcal{L}) - \Psi(\overline{\mathcal{L}}) < 0$ for $\mathcal{L} > \mathcal{L}^*$.

Finally, regarding the impact on systemic risk, notice that it depends only upon the difference in the probability of a systemic event:

$$\Delta^{SF} \equiv \left[(\Pr(B_{t+1} < B^{SC}_{CF} \mid Bounded) - (\Pr(B_{t+1} < B^{SC} \mid Original)\right].$$

Now, $B^{SC}$and B^{SC}_{CF} are increasing in the loans granted to households during period t. For levels of $\mathcal{L} > \overline{\mathcal{L}}$, the amount of time t loans is lower in the bounded distribution, so that $B^{SC}_{CF} < B^{SC}$. For levels of $\mathcal{L} \in (\underline{\mathcal{L}}, \overline{\mathcal{L}})$, L_t is the same. For levels of $\mathcal{L} < \underline{\mathcal{L}}$, the probability of a systemic crisis is zero as $\underline{\mathcal{L}}$ belongs to the U or SF regimes.

The value of B_{t+1} that occurs in the SF regime is independent of the existence of bounds and $\Delta^{SC} = 0$. For the U and L regimes, the value of the bubble is the same in both regimes if the lower bound is not reached and cannot be higher in the bounded regime if the lower bound is reached.

Consequently,

$$\left[(\Pr(B_{t+1} < B^{SC} \mid Bounded) - (\Pr(B_{t+1} < B^{SC} \mid Original)\right] \leq 0,$$

and $\Delta^{SC} \leq 0$ concluding the proof.

Proof of Proposition 5

Denote by $B(\overline{\mathcal{L}})$ and $r(\overline{\mathcal{L}})$ the corresponding values of the price of the bubble and the interest rate charged to firms for $\mathcal{L} = \overline{\mathcal{L}}$ and a given level of $\mathcal{A}$. We will use the subindexes O and B to refer to the original and the bounded distribution.

Let $\mathcal{E}_O = \int_0^{\infty} B(\mathcal{L})d\mathcal{G}(\mathcal{L})$ be the expected value of the bubble with the original distribution and $\mathcal{E}_B = \int_0^{\overline{\mathcal{L}}} B(\mathcal{L})d\mathcal{G}(\mathcal{L}) + B(\overline{\mathcal{L}}) \int_{\overline{\mathcal{L}}}^{\infty} d\mathcal{G}(\mathcal{L})$ the corresponding one with the capped distribution. For $\mathcal{L} > \overline{\mathcal{L}}$ we have $B(\mathcal{L}) > B(\overline{\mathcal{L}})$. Consequently, when the probability of the cap is non zero, the strict inequality $\mathcal{E}_B < \mathcal{E}_O$ obtains.

We will now prove that imposing a cap $\overline{\mathcal{L}}$ will reduce the riskiness of the discount factor $\frac{1}{1+r_t}$ in the sense of a mean preserving spread. To begin with, notice that taking the expected value of $\frac{\mathcal{E}}{(1+r^f)} = B$, implies $\mathbb{E}(\frac{1}{1+r_i}) = 1$, $i = B, O$, so that the two distributions have the same expected value for $\frac{1}{1+r_t}$. Define $\overline{\delta}$ as the equilibrium value of $\frac{1}{1+r_t}$ for the realization of $\overline{\mathcal{L}}$. We know that, for $\mathcal{L} < \overline{\mathcal{L}}$, conditions (12) hold, so that $1 + r_B^f(\mathcal{A}, \mathcal{L}, \mathcal{E}) = \frac{1}{2}\frac{1}{W+\mathcal{L}}\left[\mathcal{E}_B + \sqrt{\mathcal{E}_B^2 + \mathcal{A}^2 (W + \mathcal{L})}\right]$.

Still, for $\mathcal{L} \geq \overline{\mathcal{L}}$, we will have $r_B^f(\mathcal{A}, \mathcal{L}, \mathcal{E}) = r_B(\overline{\mathcal{L}})$. Consequently, because $\mathcal{E}_B < \mathcal{E}_O$, we have $r_B^f(\mathcal{A}, \overline{\mathcal{L}}, \mathcal{E}) < r_O^f(\mathcal{A}, \mathcal{L}, \mathcal{E})$. Because $\mathbb{E}(\frac{1}{1+r_i}) = 1$, for $\mathcal{L} \geq \overline{\mathcal{L}}$ this inequality has to revert, and, because of continuity, there exists a level of credit supply $\widehat{\mathcal{L}}$ such that $r_B^f(\mathcal{A}, \widehat{\mathcal{L}}, \mathcal{E}_B) = r_B(\overline{\mathcal{L}}) = r_O^f(\mathcal{A}, \widehat{\mathcal{L}}, \mathcal{E}_O)$. Therefore, for the corresponding $\widehat{\delta}$, defined as $\widehat{\delta} = \frac{1}{1+\overline{r}_B}$, we have $\Pr_B(\frac{1}{1+r_t} \leq \widehat{\delta}) = \Pr_O(\frac{1}{1+r_t} \leq \widehat{\delta})$ and the inequality reverts after this threshold. Consequently, we have the single crossing condition on the cumulative distribution function that characterize the second order stochastic dominance and the previous lemma applies.

Now, because $k = \left(\frac{\mathcal{A}}{1+r}\right)^2$, in equilibrium, the output is $\frac{\mathcal{A}^2}{1+r}$ and $F(\mathcal{A}, N, k) - k = \frac{\mathcal{A}^2}{1+r} - \left(\frac{\mathcal{A}}{1+r}\right)^2$, a concave function of $\frac{1}{1+r}$. As a consequence, a mean preserving contraction will increase the expected welfare.

Because, in addition, a cap on $\mathcal{L}$ also reduces systemic risk, it is welfare improving.

Proof of Proposition 6

Let $\overline{\mathcal{L}}$ be defined as the level of total credit supply that leads to the equilibrium amount of $\overline{L}$. For $\mathcal{L} > \overline{\mathcal{L}}$, the value of the bubble is $\overline{B} = W + \overline{L}$ in both the economy with a constraint on L and the economy with a constraint on $\mathcal{L}$. For $\mathcal{L} < \overline{\mathcal{L}}$ the equilibrium is unconstrained, so that the expected value of the bubble $\mathcal{E}$ is the same in both economies. Consequently, the difference between the two regimes stems from the allocation of capital when $\mathcal{L} > \overline{\mathcal{L}}$.

In equilibrium, for a given level of productivity, L is a non decreasing function of the credit supply. This implies the bounds on targeted credit supply will only bind for $\mathcal{L} > \overline{\mathcal{L}}$. The equilibrium is in the $L-$regime and the upper bound is binding, so that $L = \overline{L}$, implying a fixed level $\overline{B}$ for the bubble, as $\overline{B} = W + \overline{L}$. Now, because $K = \mathcal{L} - \overline{L}$, the firms' interest rate will be determined as the marginal product of capital: $\frac{\partial F(\mathcal{A},N,\mathcal{L}-\overline{L})}{\partial k} = 1 + r_{\overline{L}}^f$, while when the cap is on the credit supply, the allocation is determined by $\frac{\partial F(\mathcal{A},N,\mathcal{L}-L)}{\partial k} = 1 + r_C^f$, with $L > \overline{L}$. Consequently, $\frac{\partial F(\mathcal{A},N,\mathcal{L}-\overline{L})}{\partial k} < \frac{\partial F(\mathcal{A},N,\mathcal{L}-L)}{\partial k}$, and, by assumption, $\frac{\partial F(\mathcal{A},N,\mathcal{L}-L)}{\partial k} < 0$ for every $\mathcal{L} > \overline{\mathcal{L}}$.

To compute the difference in welfare, it is sufficient to compute

$$\Delta = \int_{\overline{\mathcal{L}}}^{\infty} \int_0^{\infty} \left[F(\mathcal{A}, N, \widetilde{K}^{\overline{\mathcal{L}}}) - \widetilde{K}^{\overline{\mathcal{L}}} - \left(F(\mathcal{A}, N, \widetilde{K}^{\overline{L}}) - \widetilde{K}^{\overline{L}}\right)\right] dH(\mathcal{A} \mid \mathcal{G}) d\mathcal{G}(\mathcal{L}),$$

where $\widetilde{K}_t^{\overline{L}}$ is the allocation of capital in the capped targeted credit supply regime and $\widetilde{K}_t^{\overline{\mathcal{L}}}$ is the corresponding allocation in the capped credit supply regime.

Let $\phi(\mathcal{A}, K) = F(\mathcal{A}, N, K) - K$. The function $\phi(\mathcal{A}, K)$ is concave in K, so $\phi'(\mathcal{A}, K)$ is decreasing. As we know that for every $\mathcal{L} > \overline{\mathcal{L}}$ we have $0 > \phi'(\mathcal{A}, K^{\overline{\mathcal{L}}}) > \phi'(\mathcal{A}, K^{\overline{L}})$. This implies $0 < K^{\overline{\mathcal{L}}} < K^{\overline{L}}$. The concavity property implies then $\frac{\phi(\mathcal{A}, K^{\overline{L}}) - \phi(\mathcal{A}, K^{\overline{\mathcal{L}}})}{K^{\overline{L}} - K^{\overline{\mathcal{L}}}} \leq \phi'(\mathcal{A}, K^{\overline{\mathcal{L}}}) < 0$, so $\phi(\mathcal{A}, K^{\overline{L}}) \leq \phi(\mathcal{A}, K^{\overline{\mathcal{L}}})$. As this is the case for every $\mathcal{A}, \Delta > 0$, the proposition holds.

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